Exercise 2.6

Q1. Product of two consecutive even numbers is 120. Find the numbers.

Solution:

Let 'x' and '$x+2$' are two consecutive even numbers.

According to the given condition,

$\Rightarrow \quad x(x+2) = 120$

$\Rightarrow \quad x^2 + 2x = 120$

$\Rightarrow \quad x^2 + 2x - 120 = 0$

$\Rightarrow \quad x^2 + 12x - 10x - 120 = 0$

$\Rightarrow \quad x(x+12) - 10(x+12) = 0$

$\Rightarrow \quad (x-10)(x+12) = 0$

$\Rightarrow \quad x - 10 = 0 \quad ; \quad x + 12 = 0$

$\Rightarrow \quad x = 10 \quad ; \quad x = -12$

The two consecutive even numbers are 10 and -12.

Q2. The difference of a positive number and its square is 380. Find the number.

Solution:

Let 'x' be a positive number.

According to the given condition,

$\Rightarrow \quad x^2 - x = 380$

$\Rightarrow \quad x^2 - x - 380 = 0$

$\Rightarrow \quad x^2 - 20x + 19x - 380 = 0$

$\Rightarrow \quad x(x-20) + 19(x-20) = 0$

$\Rightarrow \quad (x-20)(x+19) = 0$

$\Rightarrow \quad x - 20 = 0 \quad ; \quad x + 19 = 0$

$\Rightarrow \quad x = 20 \quad ; \quad x = -19$

Therefore, the number is 20.

Q3. The difference of cubes of two numbers is 91. Find them.

Solution:

Let two numbers be 'x' and '$x-1$'.

According to the given condition,

$\Rightarrow \quad (x)^3 - (x-1)^3 = 91 \quad ; \quad [(a-b)^3 = a^3 - b^3 - 3ab(a-b)]$

$\Rightarrow \quad x^3 - [x^3 - 1^3 - 3(x)(1)(x-1)] = 91$

$\Rightarrow \quad x^3 - [x^3 - 1 - 3x(x-1)] = 91$

$\Rightarrow \quad x^3 - (x^3 - 1 - 3x^2 + 3x) = 91$

$\Rightarrow \quad x^3 - x^3 + 1 + 3x^2 - 3x = 91$

$\Rightarrow \quad 3x^2 - 3x + 1 - 91 = 0$

$\Rightarrow \quad 3x^2 - 3x - 90 = 0 \qquad \Rightarrow \quad 3(x^2 - x - 30) = 0$

$\Rightarrow \quad x^2 - x - 30 = \frac{0}{3} \qquad \Rightarrow \quad x^2 - x - 30 = 0$

$\Rightarrow \quad x^2 - 6x + 5x - 30 = 0 \qquad \Rightarrow \quad x(x-6) + 5(x-6) = 0$

$\Rightarrow \quad (x-6)(x+5) = 0$

$\Rightarrow \quad x - 6 = 0 \quad ; \quad x + 5 = 0$

$\Rightarrow \quad x = 6 \quad ; \quad x = -5$

Therefore, the two numbers whose cubes differ by 91 are 6 and −5.

Q4. The sum of the Cartesian coordinates of a point is 9 and the sum of their squares is 45. Find the coordinates of the point.

Solution:

Let (x, y) be the coordinates of point.

According to the given condition,

$\Rightarrow \quad x + y = 9 \quad \ldots\ldots\ldots (i) \quad ; \quad x^2 + y^2 = 45 \quad \ldots\ldots\ldots (ii)$

From equation (i),

$\Rightarrow \quad x + y = 9$

$\Rightarrow \quad y = 9 - x \quad \ldots\ldots\ldots (iii)$

Putting $y = 9 - x$ in equation (ii).

$\Rightarrow \quad x^2 + y^2 = 45 \quad \ldots\ldots\ldots (ii)$

$\Rightarrow \quad x^2 + (9-x)^2 = 45 \qquad \Rightarrow \quad x^2 + 81 + x^2 - 18x - 45 = 0$

$\Rightarrow \quad 2x^2 - 18x + 36 = 0 \qquad \Rightarrow \quad 2(x^2 - 9x + 18) = 0$

$\Rightarrow \quad x^2 - 9x + 18 = \frac{0}{2} \qquad \Rightarrow \quad x^2 - 9x + 18 = 0$

$\Rightarrow \quad x^2 - 6x - 3x + 18 = 0 \qquad \Rightarrow \quad x(x-6) - 3(x-6) = 0$

$\Rightarrow \quad (x-3)(x-6) = 0$

$\Rightarrow \quad x - 3 = 0 \quad ; \quad x - 6 = 0$

$\Rightarrow \quad x = 3 \quad ; \quad x = 6$

Putting $x = 3$ and $x = 6$ in equation (iii).

$\Rightarrow \quad y = 9 - x \quad \ldots\ldots\ldots (iii)$

$\Rightarrow \quad x = 3 \quad ; \quad x = 6$

$\Rightarrow \quad y = 9 - 3 \quad ; \quad y = 9 - 6$

$\Rightarrow \quad y = 6 \quad ; \quad y = 3$

Therefore, the coordinates of the point are $(3,6)(6,3)$.

Q5. The sum of two numbers is 11 and the product is 30. Find the numbers.

Solution:

Let two numbers be 'x' and 'y'.

According to the given condition,

$\Rightarrow \quad x + y = 11 \quad \ldots\ldots\ldots (i)$

$\Rightarrow \quad xy = 30 \quad \ldots\ldots\ldots (ii)$

From equation (ii),

$\Rightarrow \quad xy = 30$

$\Rightarrow \quad x = \frac{30}{y}$ ……… (iii)

Putting $x = \frac{30}{y}$ in equation (i).

$\Rightarrow \quad x + y = 11$ ……… (i)

$\Rightarrow \quad \frac{30}{y} + y = 11 \quad \Rightarrow \quad \frac{30 + y^2}{y} = 11$

Multiplying by y to clear the fraction, we get:

$\Rightarrow \quad y\left(\frac{30 + y^2}{y}\right) = y(11) \quad \Rightarrow \quad y^2 + 30 = 11y$

$\Rightarrow \quad y^2 - 11y + 30 = 0 \quad \Rightarrow \quad y^2 - 5y - 6y + 30 = 0$

$\Rightarrow \quad y(y - 5) - 6(y - 5) = 0 \quad \Rightarrow \quad (y - 6)(y - 5) = 0$

$\Rightarrow \quad y - 6 = 0 \quad ; \quad y - 5 = 0$

$\Rightarrow \quad y = 6 \quad ; \quad y = 5$

Putting $y = 6$ and $y = 5$ in equation (iii).

$\Rightarrow \quad x = \frac{30}{y}$ ……… (iii)

$\Rightarrow \quad y = 6 \quad ; \quad y = 5$

$\Rightarrow \quad x = \frac{30}{6} \quad ; \quad x = \frac{30}{5}$

$\Rightarrow \quad x = 5 \quad ; \quad x = 6$

Therefore, the two numbers are 5 and 6.

Q6. The sum of the squares of two consecutive odd integers is 34. Find the integers.

Solution:

Let two consecutive odd integers be '$x - 1$' and '$x + 1$'.

According to the given condition,

$\Rightarrow \quad (x - 1)^2 + (x + 1)^2 = 34$

$\Rightarrow \quad x^2 + 1 - 2x + x^2 + 1 + 2x = 34$

$\Rightarrow \quad 2x^2 + 2 = 34 \quad \Rightarrow \quad 2x^2 = 34 - 2 \quad \Rightarrow \quad 2x^2 = 32$

$\Rightarrow \quad x^2 = \frac{32}{2} \quad \Rightarrow \quad x^2 = 16 \quad \Rightarrow \quad \sqrt{x^2} = \sqrt{16} \quad \Rightarrow \quad x = \pm 4$

Now, putting the value of x in '$x - 1$' and '$x + 1$'.

'$x - 1$' ; '$x + 1$'

- $x = 4 \quad ; \quad x = 4$

  $4 - 1 = 3 \quad ; \quad 4 + 1 = 5$

- $x = -4 \quad ; \quad x = -4$

  $-4 - 1 = -5 \quad ; \quad -4 + 1 = -3$

Hence, the integers are 3, −5, 5, −3.

Q7. The sum of ages of a father and his son is 50 years. Ten years ago, the father was 9 times as old as his son. Find the present age of father and son.

Solution:

According to the given condition,

Father	Son
$9x$	x
$9x+10$	$x+10$

Father age + Son age = 50 years

$\Rightarrow \quad 9x+10+x+10=50$

$\Rightarrow \quad 10x+20=50 \quad \Rightarrow \quad 10x=50-20$

$\Rightarrow \quad 10x=30 \quad \Rightarrow \quad x=\frac{30}{10} \quad \Rightarrow \quad x=3$

$\Rightarrow$ Father age $=9x+10=9(3)+10=27+10=37$ years

$\Rightarrow$ Son age $=x+10=3+10=13$ years

Therefore, the Sons' age = 13 and Father's age = 37.

Q8. A two-digit number is decreased by 45 when the digits are reversed. If the sum of the digits is 11, find the number.

Solution:

Let the number be '$10x+y$'.

$\Rightarrow \quad (10x+y)-(10y+x)=45 \quad \Rightarrow \quad 10x+y-10y-x=45$

$\Rightarrow \quad 9x-9y=45 \quad \Rightarrow \quad 9(x-y)=45 \quad \Rightarrow \quad x-y=\frac{45}{9}$

$\Rightarrow \quad x-y=5 \quad$ ……… (i)

By given condition,

$\Rightarrow \quad x+y=11 \quad$ ……… (ii)

Adding equation (i) and (ii).

$$\begin{array}{r} x+y=11 \\ x-y=5 \\ \hline 2x=16 \end{array}$$

$\Rightarrow \quad x=\frac{16}{2} \quad \Rightarrow \quad x=8$

Putting $x=8$ in equation (ii).

$\Rightarrow \quad x+y=11 \quad$ ……… (ii)

$\Rightarrow \quad 8+y=11 \quad \Rightarrow \quad y=11-8 \quad \Rightarrow \quad y=3$

Hence, the required number is: $10x+y=10(8)+3=80+3=83$

Q9. Sum of two numbers is 20. Find the numbers if the sum of first number and square of other is 40.

Solution:

Let two numbers be 'x' and 'y'.

According to the given condition,

$\Rightarrow \quad x + y = 20 \quad \ldots\ldots\ldots (i)$

$\Rightarrow \quad x + y^2 = 40 \quad \ldots\ldots\ldots (ii)$

From equation (i),

$\Rightarrow \quad x + y = 20$

$\Rightarrow \quad x = 20 - y \quad \ldots\ldots\ldots (iii)$

Putting $x = 20 - y$ in equation (ii).

$\Rightarrow \quad x + y^2 = 40 \quad \ldots\ldots\ldots (ii)$

$\Rightarrow \quad 20 - y + y^2 = 40 \quad \Rightarrow \quad 20 - y + y^2 - 40 = 0$

$\Rightarrow \quad y^2 - y - 20 = 0 \quad \Rightarrow \quad y^2 - 5y + 4y - 20 = 0$

$\Rightarrow \quad y(y-5) + 4(y-5) = 0$

$\Rightarrow \quad (y+4)(y-5) = 0$

$\Rightarrow \quad y + 4 = 0 \quad ; \quad y - 5 = 0$

$\Rightarrow \quad y = -4 \quad ; \quad y = 5$

Putting $y = -4$ and $y = 5$ in equation (iii).

$\Rightarrow \quad x = 20 - y \quad \ldots\ldots\ldots (iii)$

$\Rightarrow \quad y = -4 \quad ; \quad y = 5$

$\Rightarrow \quad x = 20 + 4 \quad ; \quad x = 20 - 5$

$\Rightarrow \quad x = 24 \quad ; \quad x = 15$

Therefore, the numbers are $(24, -4)(15, 5)$.

Q10. The reciprocal of the sum of reciprocals of two numbers is $\frac{12}{5}$. Find the numbers if their sum is 10.

Solution:

Let the first number be 'x'. Its reciprocal is $\frac{1}{x}$.

Let the second number be 'y'. Its reciprocal is $\frac{1}{y}$.

According to the given condition,

$\Rightarrow \quad x + y = 10 \quad \ldots\ldots\ldots (i)$

$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} = \frac{12}{5} \quad \Rightarrow \quad \frac{y+x}{xy} = \frac{12}{5}$

Multiplying by $5xy$ to clear the fraction, we get:

$\Rightarrow \quad 5xy\left(\frac{y+x}{xy}\right) = 5xy\left(\frac{12}{5}\right)$

$\Rightarrow \quad 5(y+x) = xy(12)$

$\Rightarrow \quad 5(y+x) = 12xy \quad \ldots\ldots\ldots (ii)$

From equation (i),

$\Rightarrow \quad x + y = 10$

$\Rightarrow \quad y = 10 - x \quad \ldots\ldots\ldots (iii)$

Putting $y = 10 - x$ in equation (ii).

$\Rightarrow \quad 5(y+x) = 12xy \quad \ldots\ldots\ldots (ii)$

$\Rightarrow \quad 5(10-x+x) = 12x(10-x)$

$\Rightarrow \quad 50 = 120x - 12x^2$

$\Rightarrow \quad 12x^2 - 120x + 50 = 0 \quad \Rightarrow \quad 2(6x^2 - 60x + 25) = 0$

$\Rightarrow \quad 6x^2 - 60x + 25 = \frac{0}{2} \quad \Rightarrow \quad 6x^2 - 60x + 25 = 0$

Here, $a = 6$, $b = -60$, $c = 25$

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-60) \pm \sqrt{(-60)^2 - 4(6)(25)}}{2(6)}$$

$$\Rightarrow \quad x = \frac{60 \pm \sqrt{3600 - 600}}{12} = \frac{60 \pm \sqrt{3000}}{12}$$

$$\Rightarrow \quad x = \frac{60 \pm \sqrt{100 \times 30}}{12} \quad \Rightarrow \quad x = \frac{60 \pm 10\sqrt{30}}{12}$$

$$\Rightarrow \quad x = \frac{10(6 \pm \sqrt{30})}{12} \quad \Rightarrow \quad x = \frac{5(6 \pm \sqrt{30})}{6} \quad \Rightarrow \quad x = \frac{30 \pm 5\sqrt{30}}{6}$$

Putting $x = \frac{30 \pm 5\sqrt{30}}{6}$ in equation (iii).

$\Rightarrow \quad y = 10 - x \quad \ldots\ldots\ldots (iii)$

$$\Rightarrow \quad x = \frac{30 + 5\sqrt{30}}{6} \quad ; \quad x = \frac{30 - 5\sqrt{30}}{6}$$

$$\Rightarrow \quad y = 10 - \left(\frac{30 + 5\sqrt{30}}{6}\right) \quad ; \quad y = 10 - \left(\frac{30 - 5\sqrt{30}}{6}\right)$$

$$\Rightarrow \quad y = \frac{60 - 30 - 5\sqrt{30}}{6} \quad ; \quad y = \frac{60 - 30 + 5\sqrt{30}}{6}$$

$$\Rightarrow \quad y = \frac{30 - 5\sqrt{30}}{6} \quad ; \quad y = \frac{30 + 5\sqrt{30}}{6}$$

Hence, solution set is $\left\{\left(\frac{30 + 5\sqrt{30}}{6}, \frac{30 - 5\sqrt{30}}{6}\right), \left(\frac{30 - 5\sqrt{30}}{6}, \frac{30 + 5\sqrt{30}}{6}\right)\right\}$.

Q11. A group of 1025 students form two square patterns during morning assembly. One square pattern contains 5 more students than the other. Find the number of students in each pattern.

Solution:

Total number of students $= 1025$

Let the first square pattern be 'x'.

Let the second square pattern be '$x + 5$'.

$\Rightarrow \quad (x) + (x + 5) = 1025$

$\Rightarrow \quad x + x + 5 = 1025 \quad \Rightarrow \quad 2x + 5 = 1025$

$\Rightarrow \quad 2x = 1025 - 25 \quad \Rightarrow \quad 2x = 1020 \quad \Rightarrow \quad x = \frac{1020}{2}$

$\Rightarrow \quad x = 510$

Now,

$\Rightarrow \quad x + 5 = 510 + 5 \quad ; \quad [\because \ x = 510]$

$= 515$

Therefore, there are **510** students in the first square pattern and **515** students in the second square pattern.

Q12. Sum of squares of two consecutive numbers is 145. The difference of their squares is 17. Find the numbers.

Solution:

Let two consecutive numbers be 'x' and '$x + 1$'.

According to the given condition,

$\Rightarrow \quad (x)^2 + (x+1)^2 = 145$

$\Rightarrow \quad x^2 + x^2 + 1 + 2x - 145 = 0$

$\Rightarrow \quad 2x^2 + 2x - 144 = 0 \quad \Rightarrow \quad 2(x^2 + x - 72) = 0$

$\Rightarrow \quad x^2 + x - 72 = \frac{0}{2} \quad \Rightarrow \quad x^2 + x - 72 = 0$

$\Rightarrow \quad x^2 + 9x - 8x - 72 = 0 \quad \Rightarrow \quad x(x+9) - 8(x+9) = 0$

$\Rightarrow \quad (x+9)(x-8) = 0$

$\Rightarrow \quad x + 9 = 0 \quad ; \quad x - 8 = 0$

$\Rightarrow \quad x = -9 \quad ; \quad x = 8$

Now,

$\Rightarrow \quad (x)^2 - (x+1)^2 = 17$

$\Rightarrow \quad x^2 - (x^2 + 1 + 2x) - 17 = 0$

$\Rightarrow \quad x^2 - x^2 - 1 - 2x - 17 = 0$

$\Rightarrow \quad -2x - 18 = 0 \quad \Rightarrow \quad -2x = 18 \quad \Rightarrow \quad x = \frac{-18}{2} \quad \Rightarrow \quad x = -9$

Therefore, the two consecutive numbers are -9 and 8.

Q13. A ball is thrown upwards its height $h(t)$ (in meters) after t seconds is modeled by $d(t) = -5t^2 + 20t + 2$. Find the time when the ball hit the ground.

Solution: $d(t) = -5t^2 + 20t + 2$

When the ball hits the ground, its height $d(t)$ is 0. Therefore, we need to solve for t when $d(t) = 0$.

$\Rightarrow \quad -5t^2 + 20t + 2 = 0$

We can use the quadratic formula to solve for t.

$\Rightarrow \quad t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

In this case, $a = -5$, $b = 20$, and $c = 2$. Plugging these values into the quadratic formula, we get:

$$\Rightarrow \quad t = \frac{-20 \pm \sqrt{20^2 - 4(-5)(2)}}{2(-5)} \quad \Rightarrow \quad t = \frac{-20 \pm \sqrt{400 + 40}}{-10}$$

$$\Rightarrow \quad t = \frac{-20 \pm \sqrt{440}}{-10} \quad \Rightarrow \quad t = \frac{-20 \pm 2\sqrt{110}}{-10} \quad \Rightarrow \quad t = \frac{10 \mp \sqrt{110}}{5}$$

Since time must be positive, we have two possible solutions:

$$\Rightarrow \quad t = \frac{10 - \sqrt{110}}{5} \approx \frac{10 - 10.488}{5} \approx -0.0976$$

$$\Rightarrow \quad t = \frac{10 + \sqrt{110}}{5} \approx \frac{10 + 10.488}{5} \approx 4.1$$

Since time must be positive, the solution is $t \approx 4.1$ seconds.

Q14. The stopping distance $d(x)$ (in meters) of a car traveling at x km/h is modeled by $d(x) = 0.05\,x^2 + 0.4x$. If the stopping distance is 30 meters, find the speed of the car.

Solution: $d(x) = 0.05\,x^2 + 0.4x$

We are given that the stopping distance is **30** meters, so we need to find the speed x such that $d(x) = 30$.

$$\Rightarrow \quad 0.05x^2 + 0.4x = 30$$

To solve for x, we can rearrange the equation into a quadratic equation:

$$\Rightarrow \quad 0.05x^2 + 0.4x - 30 = 0$$

To get rid of the decimal coefficients, we can multiply the entire equation by **20**.

$$\Rightarrow \quad x^2 + 8x - 600 = 0$$

$$\Rightarrow \quad x^2 + 30x - 20x - 600 = 0$$

$$\Rightarrow \quad x(x + 30) - 20(x + 30) = 0$$

$$\Rightarrow \quad (x + 30)(x - 20) = 0$$

Using the zero product property, we have two possible solutions for x.

$$\Rightarrow \quad x + 30 = 0 \quad \Rightarrow \quad x = -30$$

$$\Rightarrow \quad x - 20 = 0 \quad \Rightarrow \quad x = 20$$

Since speed must be positive, we discard the negative solution $x = -30$.
Therefore, the speed of the car is $x = 20$ km/h.

Q15. A valuable stamp 4 cm wide and 5 cm long. The stamp is to be mounted on a sheet of paper that is $5\frac{1}{2}$ times the area of the stamp. Determine the dimensions of the paper that will ensure a uniform border around the stamp.

Solution:

Let the width of the stamp be $w_s = 4$ cm and the length of the stamp be $l_s = 5$ cm.

The area of the stamp is $A_s = w_s \times l_s = 4 \times 5 = 20 \text{ cm}^2$.

The area of the paper is $5\frac{1}{2}$ times the area of the stamp, so:

$$\Rightarrow \quad A_p = 5\frac{1}{2} \times A_s = \frac{11}{2} \times 20 = 110 \text{ cm}^2$$

Let the width of the paper be w_p and the length of the paper be l_p.

Then, $A_p = w_p \times l_p = 110$.

Let the width of the uniform border around the stamp be x **cm.**

Then the width of the paper is $w_p = w_s + 2x = 4 + 2x$ and the length of the paper is $l_p = l_s + 2x = 5 + 2x$.

So we have the equation:

$$\Rightarrow \quad (4 + 2x)(5 + 2x) = 110 \quad \Rightarrow \quad 20 + 8x + 10x + 4x^2 = 110$$

$$\Rightarrow \quad 4x^2 + 18x + 20 = 110 \quad \Rightarrow \quad 4x^2 + 18x - 90 = 0$$

Dividing by 2, we get:

$$\Rightarrow \quad 2x^2 + 9x - 45 = 0$$

We can use the quadratic formula to solve for x.

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, $a = 2$, $b = 9$, and $c = -45$. Plugging these values into the quadratic formula, we get:

$$\Rightarrow \quad x = \frac{-9 \pm \sqrt{9^2 - 4(2)(-45)}}{2(2)} \quad \Rightarrow \quad x = \frac{-9 \pm \sqrt{81 + 360}}{4}$$

$$\Rightarrow \quad x = \frac{-9 \pm \sqrt{441}}{4} \quad \Rightarrow \quad x = \frac{-9 \pm 21}{4}$$

We have two possible solutions for x.

$$\Rightarrow \quad x = \frac{-9 + 21}{4} = \frac{12}{4} = 3 \quad \Rightarrow \quad x = \frac{-9 - 21}{4} = \frac{-30}{4} = -7.5$$

Since the border width x must be positive, we choose $x = 3$.

- **Dimensions of the paper:**

The width of the paper is $w_p = 4 + 2(3) = 4 + 6 = 10$ **cm.**

The length of the paper is $l_p = 5 + 2(3) = 5 + 6 = 11$ **cm.**

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