Exercise 5.3

➢ **Solve and check your solutions (1-7).**

Q1. $\dfrac{6x}{x-11}+1=\dfrac{3}{x-11}$

Solution:

We are asked to solve the equation:

$\Rightarrow \quad \dfrac{6x}{x-11}+1=\dfrac{3}{x-11}$

First, we want to get rid of the fractions.

Multiply both sides of the equation by $(x-11)$.

$\Rightarrow \quad (x-11)\left(\dfrac{6x}{x-11}+1\right)=(x-11)\left(\dfrac{3}{x-11}\right)$

$\Rightarrow \quad 6x+(x-11)=3 \quad \Rightarrow \quad 6+x-11=3$

$\Rightarrow \quad 7x-11=3 \quad \Rightarrow \quad 7x=3+11$

$\Rightarrow \quad 7x=14 \quad \Rightarrow \quad x=\dfrac{14}{7} \quad \Rightarrow \quad x=2.$

Now we check our answer:

$\Rightarrow \quad \dfrac{6x}{x-11}+1=\dfrac{3}{x-11} \quad \Rightarrow \quad \dfrac{6(2)}{2-11}+1=\dfrac{3}{2-11}$

$\Rightarrow \quad \dfrac{12}{-9}+1=\dfrac{3}{-9} \quad \Rightarrow \quad -\dfrac{4}{3}+1=-\dfrac{1}{3}$

$\Rightarrow \quad -\dfrac{4}{3}+\dfrac{3}{3}=-\dfrac{1}{3} \quad \Rightarrow \quad -\dfrac{1}{3}=-\dfrac{1}{3}$

$\therefore$ The solution $x=2$ is valid.

$\Rightarrow$ **Solution set** $=\{2\}$

Q2. $\dfrac{2y}{y+3}=\dfrac{-4}{y-7}$

Solution:

We are asked to solve the equation:

$\Rightarrow \quad \dfrac{2y}{y+3}=\dfrac{-4}{y-7}$

Cross-multiply to get rid of the fractions:

$\Rightarrow \quad 2y(y-7)=-4(y+3) \quad \Rightarrow \quad 2y^2-14y=-4y-12$

Move all terms to one side to form a quadratic equation:

$\Rightarrow \quad 2y^2-14y+4y+12=0 \quad \Rightarrow \quad 2y^2-10y+12=0$

Divide the equation by 2.

$\Rightarrow \quad y^2 - 5y + 6 = 0$

Factor the quadratic equation:

$\Rightarrow \quad (y-2)(y-3) = 0$

$\Rightarrow \quad (y-2) = 0 \; ; \quad (y-3) = 0$

So the possible solutions are $y = 2$ and $y = 3$.

Now we check the solutions:

$\Rightarrow \quad \dfrac{2y}{y+3} = \dfrac{-4}{y-7}$

- **For $y = 2$:** ; • **For $y = 3$:**

$\Rightarrow \quad \dfrac{2(2)}{2+3} = \dfrac{-4}{2-7}$; $\Rightarrow \quad \dfrac{2(3)}{3+3} = \dfrac{-4}{3-7}$

$\Rightarrow \quad \dfrac{4}{5} = \dfrac{-4}{-5}$; $\Rightarrow \quad \dfrac{6}{6} = \dfrac{-4}{-4}$

$\Rightarrow \quad \dfrac{4}{5} = \dfrac{4}{5}$; $\Rightarrow \quad 1 = 1$

So $y = 2$ is a valid solution. ; So $y = 3$ is a valid solution.

$\therefore$ The solution is valid.

$\Rightarrow$ **Solution set $= \{2, 3\}$**

Q3. $\dfrac{x+7}{x+4} - 1 = \dfrac{x+10}{2x+8}$

Solution:

We are asked to solve the equation:

$\Rightarrow \quad \dfrac{x+7}{x+4} - 1 = \dfrac{x+10}{2x+8}$

First, rewrite the equation by subtracting 1.

$\Rightarrow \quad \dfrac{x+7}{x+4} - \dfrac{x+4}{x+4} = \dfrac{x+10}{2x+8} \quad \Rightarrow \quad \dfrac{x+7-(x+4)}{x+4} = \dfrac{x+10}{2x+8}$

$\Rightarrow \quad \dfrac{x+7-x-4}{x+4} = \dfrac{x+10}{2(x+4)} \quad \Rightarrow \quad \dfrac{3}{x+4} = \dfrac{x+10}{2(x+4)}$

Multiply both sides by $2(x+4)$.

$\Rightarrow \quad 2(x+4) \times \dfrac{3}{x+4} = 2(x+4) \times \dfrac{x+10}{2(x+4)}$

$\Rightarrow \quad 6 = x + 10 \quad \Rightarrow \quad x = 6 - 10 \quad \Rightarrow \quad x = -4$

Now we check the solution. Notice that if $x = -4$, then $x + 4 = 0$, so the original equation is undefined because we would be dividing by zero.

$\Rightarrow \quad \dfrac{x+7}{x+4} - 1 = \dfrac{x+10}{2x+8} \quad \Rightarrow \quad \dfrac{-4+7}{-4+4} - 1 = \dfrac{-4+10}{2(-4)+8} \quad \Rightarrow \quad \dfrac{3}{0} - 1 = \dfrac{6}{0} = \infty$

Since we have division by zero, $x = -4$ is not a valid solution.

Therefore, there is no solution to the equation.

Q4. $\frac{3y}{y+1} = \frac{12}{y^2-1} + \frac{y+4}{y+1}$

Solution:

We are asked to solve the equation:

$\Rightarrow \quad \frac{3y}{y+1} = \frac{12}{y^2-1} + \frac{y+4}{y+1}$

First, we notice that $y^2 - 1 = (y+1)(y-1)$.

So we can rewrite the equation as:

$\Rightarrow \quad \frac{3y}{y+1} = \frac{12}{(y+1)(y-1)} + \frac{y+4}{y+1}$

Multiply both sides by $(y+1)(y-1)$ to eliminate the fractions.

$\Rightarrow \quad (y+1)(y-1)\frac{3y}{y+1} = (y+1)(y-1)\left[\frac{12}{(y+1)(y-1)} + \frac{y+4}{y+1}\right]$

$\Rightarrow \quad 3y(y-1) = 12 + (y+4)(y-1)$

$\Rightarrow \quad 3y^2 - 3y = 12 + y^2 - y + 4y - 4$

$\Rightarrow \quad 3y^2 - 3y = 12 + y^2 + 3y - 4$

$\Rightarrow \quad 3y^2 - 3y = y^2 + 3y + 8$

Move all terms to one side:

$\Rightarrow \quad 3y^2 - 3y - y^2 - 3y - 8 = 0 \quad \Rightarrow \quad 2y^2 - 6y - 8 = 0$

Divide the equation by 2.

$\Rightarrow \quad y^2 - 3y - 4 = 0$

Factor the quadratic equation:

$\Rightarrow \quad (y-4)(y+1) = 0$

So the possible solutions are $y = 4$ and $y = -1$.

Now we check the solutions:

$\Rightarrow \quad \frac{3y}{y+1} = \frac{12}{y^2-1} + \frac{y+4}{y+1}$

- **For $y = 4$:**

$\Rightarrow \quad \frac{3(4)}{4+1} = \frac{12}{4^2-1} + \frac{4+4}{4+1}$

$\Rightarrow \quad \frac{12}{5} = \frac{12}{15} + \frac{8}{5} \quad \Rightarrow \quad \frac{12}{5} = 5 + \frac{8}{5} \quad \Rightarrow \quad \frac{12}{5} = \frac{12}{5}$

So $y = 4$ is a valid solution.

- **For $y = -1$:**

$\Rightarrow \quad \frac{3(-1)}{-1+1} = \frac{12}{(-1)^2-1} + \frac{-1+4}{-1+1} \quad \Rightarrow \quad \frac{-3}{0} = \frac{12}{0} + \frac{3}{0} = \infty$

Since we have division by zero, $y = -1$ is not a valid solution.

Therefore, the only solution is $y = 4$.

$\Rightarrow$ only 4 (-1 is not solution)

Q5. $x + \frac{5}{x} = -6$

Solution:

We are asked to solve the equation:

$\Rightarrow \quad x + \frac{5}{x} = -6$

Multiply both sides of the equation by x to eliminate the fraction:

$\Rightarrow \quad x\left(x + \frac{5}{x}\right) = x(-6)$

$\Rightarrow \quad x^2 + 5 = -6x$

Move all terms to one side to form a quadratic equation:

$\Rightarrow \quad x^2 + 6x + 5 = 0$

Factor the quadratic equation:

$\Rightarrow \quad (x + 1)(x + 5) = 0$

So the possible solutions are $x = -1$ and $x = -5$.

Now we check the solutions:

- **For $x = -1$:**

$\Rightarrow \quad -1 + \frac{5}{-1} = -6$

$\Rightarrow \quad -1 - 5 = -6$

$\Rightarrow \quad -6 = -6$

So $x = -1$ is a valid solution.

- **For $x = -5$:**

$\Rightarrow \quad -5 + \frac{5}{-5} = -6$

$\Rightarrow \quad -5 - 1 = -6$

$\Rightarrow \quad -6 = -6$

So $x = -5$ is a valid solution.

$\Rightarrow \quad$ **Solution set** $= \{-1, -5\}$

Q6. $\frac{y+2}{y^2 + 6y - 7} = \frac{8}{y^2 + 3y - 4}$

Solution:

We are asked to solve the equation:

$\Rightarrow \quad \frac{y+2}{y^2 + 6y - 7} = \frac{8}{y^2 + 3y - 4}$

First, factor the denominators:

$\Rightarrow \quad y^2 + 6y - 7 = (y + 7)(y - 1)$

$\Rightarrow \quad y^2 + 3y - 4 = (y + 4)(y - 1)$

Rewrite the equation with factored denominators:

$\Rightarrow \quad \frac{y+2}{(y+7)(y-1)} = \frac{8}{(y+4)(y-1)}$

Multiply both sides by $(y + 7)(y - 1)(y + 4)$ to eliminate the fractions.

$\Rightarrow \quad (y+7)(y-1)(y+4)\frac{y+2}{(y+7)(y-1)} = (y+7)(y-1)(y+4)\frac{8}{(y+4)(y-1)}$

$\Rightarrow \quad (y + 4)(y + 2) = 8(y + 7)$

$\Rightarrow \quad y^2 + 2y + 4y + 8 = 8y + 56$

$\Rightarrow \quad y^2 + 6y + 8 = 8y + 56$

Move all terms to one side to form a quadratic equation:

$\Rightarrow \quad y^2 + 6y - 8y + 8 - 56 = 0$

$\Rightarrow \quad y^2 - 2y - 48 = 0$

Factor the quadratic equation:

$\Rightarrow \quad (y - 8)(y + 6) = 0$

So the possible solutions are $y = 8$ and $y = -6$.

Now we check the solutions:

$$\Rightarrow \quad \frac{y + 2}{y^2 + 6y - 7} = \frac{8}{y^2 + 3y - 4}$$

- **For $y = 8$:**

$$\Rightarrow \quad \frac{8 + 2}{8^2 + 6(8) - 7} = \frac{8}{8^2 + 3(8) - 4}$$

$$\Rightarrow \quad \frac{10}{64 + 48 - 7} = \frac{8}{64 + 24 - 4} \quad \Rightarrow \quad \frac{10}{105} = \frac{8}{84} \quad \Rightarrow \quad \frac{2}{21} = \frac{2}{21}$$

So $y = 8$ is a valid solution.

- **For $y = -6$:**

$$\Rightarrow \quad \frac{-6 + 2}{(-6)^2 + 6(-6) - 7} = \frac{8}{(-6)^2 + 3(-6) - 4}$$

$$\Rightarrow \quad \frac{-4}{36 - 36 - 7} = \frac{8}{36 - 18 - 4} \quad \Rightarrow \quad \frac{-4}{-7} = \frac{8}{14} \quad \Rightarrow \quad \frac{4}{7} = \frac{4}{7}$$

So $y = -6$ is a valid solution.

$\Rightarrow$ **Solution set** $= \{-6, 8\}$

Q7. $\frac{5}{y + 1} + \frac{3y + 5}{y^2 + 4y + 3} = \frac{2}{y + 3}$

Solution:

We are asked to solve the equation:

$$\Rightarrow \quad \frac{5}{y + 1} + \frac{3y + 5}{y^2 + 4y + 3} = \frac{2}{y + 3}$$

First, factor the denominator $y^2 + 4y + 3 = (y + 1)(y + 3)$.

Rewrite the equation with the factored denominator:

$$\Rightarrow \quad \frac{5}{y + 1} + \frac{3y + 5}{(y + 1)(y + 3)} = \frac{2}{y + 3}$$

Multiply both sides by $(y + 1)(y + 3)$ to eliminate the fractions.

$$\Rightarrow \quad (y + 1)(y + 3)\left(\frac{5}{y + 1} + \frac{3y + 5}{(y + 1)(y + 3)}\right) = (y + 1)(y + 3)\,\frac{2}{y + 3}$$

$\Rightarrow \quad 5(y + 3) + (3y + 5) = 2(y + 1)$

$\Rightarrow$ $5y + 15 + 3y + 5 = 2y + 2$

$\Rightarrow$ $8y + 20 = 2y + 2$

Move all terms with y to one side and constants to the other side:

$\Rightarrow$ $8y - 2y = 2 - 20 \quad \Rightarrow \quad 6y = -18$

$\Rightarrow$ $y = \frac{-18}{6} \quad \Rightarrow \quad y = -3$

Now we check the solution:

$$\Rightarrow \quad \frac{5}{y+1} + \frac{3y+5}{y^2+4y+3} = \frac{2}{y+3}$$

If $y = -3$, then $y + 3 = 0$, so the original equation is undefined because we would be dividing by zero.

$$\Rightarrow \quad \frac{5}{-3+1} + \frac{3(-3)+5}{(-3)^2+4(-3)+3} = \frac{2}{-3+3}$$

$$\Rightarrow \quad \frac{5}{-2} + \frac{-9+5}{9-12+3} = \frac{2}{0} \quad \Rightarrow \quad \frac{-5}{2} + \frac{-4}{0} = \frac{2}{0} = \infty$$

Since we have division by zero, $y = -3$ is not a valid solution.

Also, if $y = -1$, then $y + 1 = 0$, so $\frac{5}{y+1}$ is undefined.

Therefore, there is **no solution.**

Q8. Kaleem can mow a lawn in 4 hours. Moiz can mow the same lawn in 5 hours. How long would it take both of them, working together, to mow the lawn.

Solution:

Let K be the rate at which Kaleem mows the lawn, and M be the rate at which Moiz mows the lawn.

Kaleem can mow a lawn in 4 hours, so $K = \frac{1}{4}$ lawns per hour.

Moiz can mow the same lawn in 5 hours, so $M = \frac{1}{5}$ lawns per hour.

When they work together, their rates add up. So their combined rate is:

$$\Rightarrow \quad K + M = \frac{1}{4} + \frac{1}{5} = \frac{5}{20} + \frac{4}{20} = \frac{9}{20}$$

So together they mow $\frac{9}{20}$ lawns per hour. To find the time it takes for them to mow one lawn together, we take the reciprocal of their combined *rate:*

$$\Rightarrow \quad \text{Time} = \frac{1}{\frac{9}{20}} = \frac{20}{9}$$

To express this as hours and minutes, we have:

$$\Rightarrow \quad \frac{20}{9} = 2\frac{2}{9} \text{ hours}$$

Now, convert the fraction of an hour to minutes:

$\Rightarrow \quad \frac{2}{9}\text{ hours} \times 60\,\frac{\text{minutes}}{\text{hour}} = \frac{120}{9}\text{ minutes} = \frac{40}{3}\text{ minutes} \approx 13\text{ minutes}$

So it would take them 2 hours and $13\frac{1}{3}$ minutes.

In decimal form, $\frac{20}{9} \approx 2$ hours.

$\Rightarrow$ Approximately $2\frac{2}{9}$ or about 2 **hours** and 13 **minutes**

Q9. You have an 8-pint mixture of paint that is made up of equal amounts of yellow paint and blue paint. To create a certain shade of green, you need a paint mixture that is 80% yellow. How many pints of yellow paint do you need to add to the mixture?

Solution:

Let y be the amount of yellow paint to add to the mixture.

The mixture is 8 pints and is made up of equal amounts of yellow and blue paint, so there are 4 pints of yellow paint and 4 pints of blue paint.

When we add y pints of yellow paint, the amount of yellow paint becomes $4 + y$ pints.

The total volume of the mixture becomes $8 + y$ pints.

We want the new mixture to be 80% yellow, so we set up the equation:

$\Rightarrow \quad \frac{4+y}{8+y} = 0.8$

Multiply both sides by $8 + y$.

$\Rightarrow \quad 4 + y = 0.8(8 + y)$

$\Rightarrow \quad 4 + y = 6.4 + 0.8y$

Subtract $0.8y$ from both sides.

$\Rightarrow \quad 4 + 0.2y = 6.4$

Subtract 4 from both sides.

$\Rightarrow \quad 0.2y = 2.4$

Divide by 0.2. $\quad \Rightarrow \quad y = \frac{2.4}{0.2} = \frac{24}{2} = 12$

So, we need to add 12 pints of yellow paint.

- **Check:**

New amount of yellow paint: $4 + 12 = 16$ pints

Total volume of mixture: $8 + 12 = 20$ pints

Percentage of yellow paint: $\frac{16}{20} = \frac{4}{5} = 0.8 = 80\%$

$\Rightarrow$ Add 12 pints of yellow paint to the mixture to achieve a paint mixture that is 80% yellow.

Q10. Waqar takes 9 hours longer to build a wall than it takes Wasi. If they work together, they can build the wall in 20 hours. How long would it take each, working alone, to build the wall?

Solution:

Let w be the time it takes Wasi to build the wall alone.

Then Waqar takes $w + 9$ hours to build the wall alone.

- Wasi's rate of work is $\frac{1}{w}$ walls per hour.
- Waqar's rate of work is $\frac{1}{w+9}$ walls per hour.

When they work together, their combined rate is $\frac{1}{w} + \frac{1}{w+9}$ walls per hour.

They can build the wall in 20 hours working together, so their combined rate is $\frac{1}{20}$ walls per hour.

Therefore, we have the equation: $\frac{1}{w} + \frac{1}{w+9} = \frac{1}{20}$

Multiply both sides by $20w(w+9)$.

$\Rightarrow$ $20(w+9) + 20w = w(w+9)$

$\Rightarrow$ $20w + 180 + 20w = w^2 + 9w$

$\Rightarrow$ $40w + 180 = w^2 + 9w$

$\Rightarrow$ $w^2 - 31w - 180 = 0$

We are looking for two numbers that multiply to -180 and add to -31.

These are -36 and 5.

$\Rightarrow$ $(w - 36)(w + 5) = 0$

So, $w = 36$ or $w = -5$. Since time cannot be negative, we have $w = 36$.

$\Rightarrow$ Wasi takes 36 **hours** to build the wall alone.

$\Rightarrow$ Waqar takes $36 + 9 = 45$ **hours** to build the wall alone.

- **Check:**

Wasi's rate: $\frac{1}{36}$; **Waqar's rate:** $\frac{1}{45}$

Combined rate: $\frac{1}{36} + \frac{1}{45} = \frac{5}{180} + \frac{4}{180} = \frac{9}{180} = \frac{1}{20}$

So together they take 20 **hours.**

$\Rightarrow$ Wasi takes 36 **hours** and Waqar takes 45.

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