Exercise 5.2

Q1. Add the following expressions.

(i) $\frac{x}{2}, \frac{x}{5}$

Solution: $\frac{x}{2}, \frac{x}{5}$

To add the expressions $\frac{x}{2}$ and $\frac{x}{5}$, we need to find a common denominator. The least common multiple of **2** and **5** is **10**.

So we rewrite the fractions with a denominator of **10**.

$\Rightarrow \quad \frac{x}{2} = \frac{x \times 5}{2 \times 5} = \frac{5x}{10}$

$\Rightarrow \quad \frac{x}{5} = \frac{x \times 2}{5 \times 2} = \frac{2x}{10}$

Now, we add the fractions:

$\Rightarrow \quad \frac{5x}{10} + \frac{2x}{10} = \frac{5x + 2x}{10} = \frac{7x}{10}$

So, the sum of the expressions is $\frac{7x}{10}$.

(ii) $\frac{x-2}{2}, \frac{x+10}{9}$

Solution:

To add the expressions $\frac{x-2}{2}$ and $\frac{x+10}{9}$, we need to find a common denominator. The least common multiple of **2** and **9** is **18**.

So we rewrite the fractions with a denominator of **18**.

$\Rightarrow \quad \frac{x-2}{2} = \frac{(x-2) \times 9}{2 \times 9} = \frac{9(x-2)}{18} = \frac{9x-18}{18}$

$\Rightarrow \quad \frac{x+10}{9} = \frac{(x+10) \times 2}{9 \times 2} = \frac{2(x+10)}{18} = \frac{2x+20}{18}$

Now, we add the fractions:

$\Rightarrow \quad \frac{9x-18}{18} + \frac{2x+20}{18} = \frac{(9x-18)+(2x+20)}{18} = \frac{9x-18+2x+20}{18}$

Combine like terms in the numerator:

$\Rightarrow \quad \frac{9x+2x-18+20}{18} = \frac{11x+2}{18}$

So, the sum of the expressions is $\frac{11x+2}{18}$.

(iii) $\frac{4+x}{4}, \frac{x-1}{7}, \frac{5x}{2}$

Solution:

To add the expressions $\frac{4+x}{4}$, $\frac{x-1}{7}$, and $\frac{5x}{2}$, we need to find a common denominator. The least common multiple of $4, 7,$ and 2 is 28.

So we rewrite the fractions with a denominator of $28.$

$$\Rightarrow \quad \frac{4+x}{4} = \frac{(4+x)\times 7}{4\times 7} = \frac{7(4+x)}{28} = \frac{28+7x}{28}$$

$$\Rightarrow \quad \frac{x-1}{7} = \frac{(x-1)\times 4}{7\times 4} = \frac{4(x-1)}{28} = \frac{4x-4}{28}$$

$$\Rightarrow \quad \frac{5x}{2} = \frac{5x\times 14}{2\times 14} = \frac{70x}{28}$$

Now, we add the fractions:

$$\Rightarrow \quad \frac{28+7x}{28} + \frac{4x-4}{28} + \frac{70x}{28} = \frac{(28+7x)+(4x-4)+(70x)}{28}$$

Combine like terms in the numerator:

$$\Rightarrow \quad \frac{28+7x+4x-4+70x}{28} = \frac{7x+4x+70x+28-4}{28} = \frac{81x+24}{28}$$

So, the sum of the expressions is $\frac{81x+24}{28}$.

(iv) $\frac{3x}{x+5}, \frac{10}{5x+25}$

Solution:

To add the expressions $\frac{3x}{x+5}$ and $\frac{10}{5x+25}$, we need to find a common denominator. First, we can factor the denominator of the second expression:

$$\Rightarrow \quad 5x+25 = 5(x+5)$$

So, the expressions are: $\frac{3x}{x+5}$ and $\frac{10}{5(x+5)}$

The least common multiple of $x+5$ and $5(x+5)$ is $5(x+5)$. Now we rewrite the first fraction with the common denominator:

$$\Rightarrow \quad \frac{3x}{x+5} = \frac{3x\times 5}{(x+5)\times 5} = \frac{15x}{5(x+5)}$$

Now, we add the fractions:

$$\Rightarrow \quad \frac{15x}{5(x+5)} + \frac{10}{5(x+5)} = \frac{15x+10}{5(x+5)}$$

We can factor the numerator: $15x+10 = 5(3x+2)$

So, the expression becomes: $\frac{5(3x+2)}{5(x+5)}$

Now, we can cancel out the common factor of 5. $\Rightarrow \dfrac{3x+2}{x+5}$

So, the sum of the expressions is $\dfrac{3x+2}{x+5}$.

(v) $\dfrac{24x}{6x-18}, \dfrac{3(1+x)}{x-3}$

Solution:

To add the expressions $\dfrac{24x}{6x-18}$ and $\dfrac{3(1+x)}{x-3}$, we need to find a common denominator. First, we can factor the denominator of the first expression:

$\Rightarrow \quad 6x-18=6(x-3)$

So, the expressions are: $\dfrac{24x}{6(x-3)}$ and $\dfrac{3(1+x)}{x-3}$

Simplify the first fraction: $\dfrac{24x}{6(x-3)}=\dfrac{4x}{x-3}$

Now, we have: $\dfrac{4x}{x-3}$ and $\dfrac{3(1+x)}{x-3}$

The common denominator is $x-3$. Now we add the fractions:

$\Rightarrow \quad \dfrac{4x}{x-3}+\dfrac{3(1+x)}{x-3}=\dfrac{4x+3(1+x)}{x-3}$

Expand the numerator: $\dfrac{4x+3+3x}{x-3}$

Combine like terms in the numerator: $\dfrac{7x+3}{x-3}$

So, the sum of the expressions is $\dfrac{7x+3}{x-3}$.

Q2. <u>Subtract.</u>

(i) $\dfrac{23-x}{5}$ **from 7**

Solution:

To subtract $\dfrac{23-x}{5}$ from 7, we write the expression as: $7-\dfrac{23-x}{5}$

We need to find a common denominator, which is 5.

So, we rewrite 7 with a denominator of 5.

$\Rightarrow \quad 7=\dfrac{7\times 5}{5}=\dfrac{35}{5}$

Now, we subtract the fractions: $\dfrac{35}{5}-\dfrac{23-x}{5}=\dfrac{35-(23-x)}{5}$

Distribute the negative sign in the numerator: $\dfrac{35-23+x}{5}$

Combine like terms in the numerator: $\dfrac{12+x}{5}$

So, the result is $\frac{x+12}{5}$.

(ii) $\frac{6(x-8)}{7}$ **from** $\frac{5(x-7)}{3}$

Solution:

To subtract $\frac{6(x-8)}{7}$ from $\frac{5(x-7)}{3}$, we write the expression as:

$$\Rightarrow \quad \frac{5(x-7)}{3} - \frac{6(x-8)}{7}$$

We need to find a common denominator, which is the least common multiple of 3 and 7, which is **21.**

So we rewrite the fractions with a denominator of **21.**

$$\Rightarrow \quad \frac{5(x-7)}{3} = \frac{5(x-7)\times 7}{3\times 7} = \frac{35(x-7)}{21} = \frac{35x-245}{21}$$

$$\Rightarrow \quad \frac{6(x-8)}{7} = \frac{6(x-8)\times 3}{7\times 3} = \frac{18(x-8)}{21} = \frac{18x-144}{21}$$

Now, we subtract the fractions:

$$\Rightarrow \quad \frac{35x-245}{21} - \frac{18x-144}{21} = \frac{(35x-245)-(18x-144)}{21}$$

Distribute the negative sign in the numerator: $\frac{35x-245-18x+144}{21}$

Combine like terms in the numerator: $\frac{35x-18x-245+144}{21} = \frac{17x-101}{21}$

So, the result is $\frac{17x-101}{21}$.

(iii) $2x^2-2x+1$ **from** $\frac{x+1}{x}$

Solution:

$$\Rightarrow \quad \frac{x+1}{x} - (2x^2-2x+1)$$

First, we can rewrite $\frac{x+1}{x}$ as $\frac{x}{x}+\frac{1}{x} = 1+\frac{1}{x}$.

So the expression becomes:

$$= 1+\frac{1}{x} - (2x^2-2x+1)$$

Distribute the negative sign:

$$= 1+\frac{1}{x} - 2x^2 + 2 - 1 = 2x - 2x^2 + \frac{1}{x}$$

To combine these terms, we can find a common denominator, which is x.

$$\Rightarrow \quad \frac{2x^2}{x} - \frac{2x^3}{x} + \frac{1}{x} = \frac{2x^2-2x^3+1}{x}$$

We can also write this as $\frac{-2x^3+2x^2+1}{x}$.

Q3. Divide the first expression by the second.

(i) $x^4 - 10x^2 + 9$, $x^2 - 2x - 3$

Solution: $x^4 - 10x^2 + 9 \div x^2 - 2x - 3$

$\Rightarrow \quad \dfrac{x^4 - 10x^2 + 9}{x^2 - 2x - 3} = \dfrac{(x-1)(x+1)(x-3)(x+3)}{(x-3)(x+1)}$

The problem asks us to divide $x^4 - 10x^2 + 9$ by $x^2 - 2x - 3$.

First, we can factor both expressions.

$\Rightarrow$ $x^4 - 10x^2 + 9$ can be factored as a quadratic in x^2.

Let $y = x^2$. Then we have $y^2 - 10y + 9$. This factors as $(y-1)(y-9)$.

Substituting x^2 back in for y, we have $(x^2 - 1)(x^2 - 9)$.

These are both differences of squares, so we can factor further:

$\Rightarrow$ $x^2 - 1 = (x-1)(x+1)$ and $x^2 - 9 = (x-3)(x+3)$

So, $x^4 - 10x^2 + 9 = (x-1)(x+1)(x-3)(x+3)$.

Next, we factor $x^2 - 2x - 3$.

We look for two numbers that multiply to -3 and add to -2. These numbers are -3 and 1.

So, $x^2 - 2x - 3 = (x-3)(x+1)$.

Now, we divide the first expression by the second:

$\Rightarrow \quad \dfrac{x^4 - 10x^2 + 9}{x^2 - 2x - 3} = \dfrac{(x-1)(x+1)(x-3)(x+3)}{(x-3)(x+1)}$

We can cancel the common factors of $(x+1)$ and $(x-3)$, assuming $x \neq -1$ and $x \neq 3$.

This leaves us with $(x-1)(x+3)$.

Expanding this, we get $x^2 + 3x - x - 3 = x^2 + 2x - 3$.

Thus, the result of dividing $x^4 - 10x^2 + 9$ by $x^2 - 2x - 3$ is $x^2 + 2x - 3$.

The final answer is $x^2 + 2x - 3$.

(ii) $x^3 - 3x^2y + 3xy^2 - y^3$, $x - y$

Solution:

$\Rightarrow \quad \dfrac{x^3 - 3x^2y + 3xy^2 - y^3}{x-y} = \dfrac{(x-y)^3}{x-y} = (x-y)^2$

Expanding $(x-y)^2$, we get:

$\Rightarrow \quad (x-y)(x-y) = x^2 - xy - xy + y^2 = x^2 - 2xy + y^2$

(iii) $\dfrac{4x^2 - 16}{5x}$, $\dfrac{2x+4}{15}$

Solution:

$\Rightarrow \quad \dfrac{4x^2 - 16}{5x} \div \dfrac{2x+4}{15} = \dfrac{4x^2 - 16}{5x} \times \dfrac{15}{2x+4}$

We can factor $4x^2 - 16$ as $4(x^2 - 4) = 4(x-2)(x+2)$.

We can factor $2x + 4$ as $2(x+2)$.

So the expression becomes:

$$\Rightarrow \quad \frac{4(x-2)(x+2)}{5x} \times \frac{15}{2(x+2)}$$

$$\Rightarrow \quad \frac{2(x-2) \times 3}{x} = \frac{6(x-2)}{x} = \frac{6x-12}{x} = \frac{6(x-2)}{x}$$

(iv) $\frac{x^2+5x}{x-3}, \quad \frac{x^2-25}{x-3}$

Solution:

$$\Rightarrow \quad \frac{x^2+5x}{x-3} \div \frac{x^2-25}{x-3} = \frac{x^2+5x}{x-3} \times \frac{x-3}{x^2-25}$$

$$= \frac{x(x+5)}{x-3} \times \frac{x-3}{(x-5)(x+5)} = \frac{x}{x-5}$$

Q4. <u>Simplify the following.</u>

(i) $\frac{x}{2} + \frac{x}{3} - \frac{x}{4} + \frac{x}{5}$

Solution:

We can factor out x from each term:

$$= x\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}\right)$$

Now, we need to find a common denominator for the fractions. The least common multiple of $2, 3, 4, 5$ is 60.

$$= x\left(\frac{30+20-15+12}{60}\right) = x\left(\frac{47}{60}\right) = \frac{47x}{60}$$

(ii) $\frac{1}{2}\left(4 - \frac{x}{3}\right) - \frac{5}{6} + \frac{1}{3}\left(11 - \frac{x}{2}\right)$

Solution:

First, distribute the $\frac{1}{2}$ and $\frac{1}{3}$.

$$\Rightarrow \quad \frac{1}{2}(4) - \frac{1}{2}\left(\frac{x}{3}\right) - \frac{5}{6} + \frac{1}{3}(11) - \frac{1}{3}\left(\frac{x}{2}\right) = 2 - \frac{x}{6} - \frac{5}{6} + \frac{11}{3} - \frac{x}{6}$$

Now, combine the constant terms and the x terms:

$$\Rightarrow \quad 2 - \frac{5}{6} + \frac{11}{3} - \frac{x}{6} - \frac{x}{6} = 2 - \frac{5}{6} + \frac{22}{6} - \frac{2x}{6} = 2 + \frac{17}{6} - \frac{x}{3}$$

To combine these terms, we can find a common denominator, which is 6.

$$= \frac{12+17-2x}{6} = \frac{29-2x}{6}$$

(iii) $\frac{2}{x+1}+\frac{x}{x-1}-\frac{x+2}{x-1}$

Solution:

$$\Rightarrow \quad \frac{2}{x+1}+\frac{x}{x-1}-\frac{x+2}{x-1}$$

First, we can combine the last two terms since they have the same denominator:

$$\Rightarrow \quad \frac{2}{x+1}+\frac{x-(x+2)}{x-1}=\frac{2}{x+1}+\frac{x-x-2}{x-1}=\frac{2}{x+1}+\frac{-2}{x-1}=\frac{2}{x+1}-\frac{2}{x-1}$$

$$=\frac{2(x-1)-2(x+1)}{(x+1)(x-1)}=\frac{2x-2-2x-2}{(x+1)(x-1)}=\frac{-4}{x^2-1}$$

(iv) $\frac{x^2-25}{5}-\frac{x}{4}\div\frac{3x}{20}$

Solution:

$$\Rightarrow \quad \frac{x^2-25}{5}-\frac{x}{4}\div\frac{3x}{20}$$

First, we rewrite the division as multiplication:

$$=\frac{x^2-25}{5}-\frac{x}{4}\times\frac{20}{3x}=\frac{x^2-25}{5}-\frac{5}{3}$$

$$=\frac{3x^2-75-25}{15}=\frac{3x^2-100}{15}$$

(v) $\frac{45a^2b^3c^4}{27x^4y^3z}\times\frac{243xy^2z^3}{180a^2bc^3}$

Solution: $\frac{45a^2b^3c^4}{27x^4y^3z}\times\frac{243xy^2z^3}{180a^2bc^3}$

First, we can write this as a single fraction:

$$\Rightarrow \quad \frac{45a^2b^3c^4\cdot 243xy^2z^3}{27x^4y^3z\cdot 180a^2bc^3}$$

Now, let's simplify the coefficients:

$$\Rightarrow \quad \frac{45\cdot 243}{27\cdot 180}=\frac{45\cdot(9\cdot 27)}{27\cdot(4\cdot 45)}=\frac{9}{4}$$

Now, let's simplify the variables:

$$\Rightarrow \quad \frac{a^2}{a^2}=1,\quad \frac{b^3}{b}=b^2,\quad \frac{c^4}{c^3}=c,\quad \frac{x}{x^4}=\frac{1}{x^3},\quad \frac{y^2}{y^3}=\frac{1}{y},\quad \frac{z^3}{z}=z^2$$

Putting it all together, we have:

$$\Rightarrow \quad \frac{9}{4}\cdot\frac{b^2c}{x^3y}\cdot z^2=\frac{9b^2cz^2}{4x^3y}$$

(vi) $\frac{m^2}{8n} \times \frac{36p^3q^2}{81mn} \div \frac{15mpx^5}{270n^2x^3y}$

Solution:

First, we rewrite the division as multiplication by the reciprocal:

$\Rightarrow \quad \frac{m^2}{8n} \times \frac{36p^3q^2}{81mn} \times \frac{270n^2x^3y}{15mpx^5}$

Now, we can write this as a single fraction:

$\Rightarrow \quad \frac{m^2 \cdot 36p^3q^2 \cdot 270n^2x^3y}{8n \cdot 81mn \cdot 15mpx^5}$

Simplify the coefficients:

$\Rightarrow \quad \frac{36 \cdot 270}{8 \cdot 81 \cdot 15} = \frac{(4 \cdot 9) \cdot (18 \cdot 15)}{8 \cdot (9 \cdot 9) \cdot 15} = \frac{4 \cdot 18}{8 \cdot 9} = \frac{4 \cdot (2 \cdot 9)}{8 \cdot 9} = \frac{8}{8} = 1$

However, let's do this in a slightly different way to avoid errors:

$\Rightarrow \quad \frac{36 \cdot 270}{8 \cdot 81 \cdot 15} = \frac{36 \cdot (18 \cdot 15)}{8 \cdot 81 \cdot 15} = \frac{36 \cdot 18}{8 \cdot 81} = \frac{(4 \cdot 9) \cdot (2 \cdot 9)}{8 \cdot (9 \cdot 9)} = \frac{4 \cdot 2}{8} = \frac{8}{8} = 1$

Now, let's simplify the variables:

$\Rightarrow \quad \frac{m^2}{m \cdot m} = 1, \ \frac{p^3}{p} = p^2, \ \ q^2 = q^2, \ \ \frac{n^2}{n \cdot n} = 1, \ \frac{x^3}{x^5} = \frac{1}{x^2}, \ \ y = y$

Putting it all together, we have:

$= \frac{p^2q^2y}{x^2}$

(vii) $3x \div \frac{3x^2 - 27}{x + 3} + \frac{1}{x - 3}$

Solution:

First, we rewrite the division as multiplication by the reciprocal:

$\Rightarrow \quad 3x \times \frac{x + 3}{3x^2 - 27} + \frac{1}{x - 3}$

We can factor $3x^2 - 27$ as $3(x^2 - 9) = 3(x - 3)(x + 3)$.

So the expression becomes:

$= 3 \times \frac{x + 3}{3(x - 3)(x + 3)} + \frac{1}{x - 3} = \frac{x}{x - 3} + \frac{1}{x - 3}$

Now, we have a common denominator, so we can add the fractions:

$= \frac{x + 1}{x - 3}$

(viii) $\frac{5x + 5}{3(2x - 1)} + \frac{6 - 2x}{2(1 - 2x)}$

Solution: $\frac{5x + 5}{3(2x - 1)} + \frac{6 - 2x}{2(1 - 2x)}$

Notice that $1 - 2x = -(2x - 1)$.

So we can rewrite the second term as:

$$\Rightarrow \quad \frac{6-2x}{2(1-2x)} = \frac{6-2x}{-2(2x-1)} = -\frac{6-2x}{2(2x-1)} = \frac{2x-6}{2(2x-1)}$$

So the expression becomes:

$$= \frac{5x+5}{3(2x-1)} + \frac{2x-6}{2(2x-1)}$$

Now, we need to find a common denominator.

The common denominator is $6(2x-1)$.

$$\Rightarrow \quad \frac{2(5x+5)}{6(2x-1)} + \frac{3(2x-6)}{6(2x-1)} = \frac{10x+10}{6(2x-1)} + \frac{6x-18}{6(2x-1)}$$

$$= \frac{10x+10+6x-18}{6(2x-1)} = \frac{16x-8}{6(2x-1)}$$

We can factor out an 8 from the numerator.

$$= \frac{8(2x-1)}{6(2x-1)} = \frac{8}{6} = \frac{4}{3}$$

Thus, the simplified expression is $\frac{4}{3}$.

(ix) $\frac{2a}{2a-3} - \frac{5}{6a+9} - \frac{4(3a+2)}{3(4a^2-9)}$

Solution:

We are asked to simplify the expression:

$$\Rightarrow \quad \frac{2a}{2a-3} - \frac{5}{6a+9} - \frac{4(3a+2)}{3(4a^2-9)}$$

$$\Rightarrow \quad \frac{2a}{2a-3} - \frac{5}{3(2a+3)} - \frac{4(3a+2)}{3(2a-3)(2a+3)}$$

Now, we need to find a common denominator, which is $3(2a-3)(2a+3)$.

$$\Rightarrow \quad \frac{2a\cdot 3(2a+3)}{3(2a-3)(2a+3)} - \frac{5\cdot(2a-3)}{3(2a-3)(2a+3)} - \frac{4(3a+2)}{3(2a-3)(2a+3)}$$

So we have:

$$= \frac{6a(2a+3)-5(2a-3)-4(3a+2)}{3(2a-3)(2a+3)}$$

Now, let's simplify the numerator:

$$6a(2a+3)-5(2a-3)-4(3a+2)$$
$$= 12a^2+18a-(10a-15)-(12a+8)$$
$$= 12a^2+18a-10a+15-12a-8$$
$$= 12a^2+(18-10-12)a+(15-8)$$
$$= 12a^2-4a+7$$

So the expression becomes:

$$\Rightarrow \quad \frac{12a^2-4a+7}{3(2a-3)(2a+3)} = \frac{12a^2-4a+7}{3(4a^2-9)} = \frac{12a^2-4a+7}{12a^2-27} = \frac{12a^2-4a+7}{3(4a^2-9)}$$

(x) $\frac{5}{5+x-18x^2} - \frac{2}{2+5x+2x^2}$

Solution:

Let's factor the denominators first.

$$\Rightarrow \quad 5+x-18x^2 = -(18x^2-x-5) = -(2x+1)(9x-5)$$
$$= (5-9x)(2x+1)$$
$$\Rightarrow \quad 2+5x+2x^2 = (2x+1)(x+2)$$

Therefore, the expression can be rewritten as:

$$= \frac{5}{(5-9x)(2x+1)} - \frac{2}{(2x+1)(x+2)}$$

The common denominator is $(5-9x)(2x+1)(x+2)$.

$$= \frac{5(x+2)}{(5-9x)(2x+1)(x+2)} - \frac{2(5-9x)}{(5-9x)(2x+1)(x+2)}$$

Combining the fractions, we have:

$$\Rightarrow \quad \frac{5(x+2)-2(5-9x)}{(5-9x)(2x+1)(x+2)} = \frac{5x+10-10+18x}{(5-9x)(2x+1)(x+2)} = \frac{23x}{(1+2x)(2+x)(5-9x)}$$

(xi) $\frac{m+3}{24m} - \frac{m+1}{24m} + \frac{3m-1}{6m^2+18m} \div \frac{12m-4}{m+3}$

Solution:

We are asked to simplify the expression:

$$\Rightarrow \quad \frac{m+3}{24m} - \frac{m+1}{24m} + \frac{3m-1}{6m^2+18m} \div \frac{12m-4}{m+3}$$

First, let's simplify the first two terms since they have a common denominator:

$$\Rightarrow \quad \frac{m+3}{24m} - \frac{m+1}{24m} = \frac{(m+3)-(m+1)}{24m} = \frac{m+3-m-1}{24m} = \frac{2}{24m} = \frac{1}{12m} \quad \ldots\ldots (1)$$

Now let's simplify the third term, starting with factoring:

$$\Rightarrow \quad 6m^2+18m = 6m(m+3) \quad \Rightarrow \quad 12m-4 = 4(3m-1)$$

So the third term is:

$$\Rightarrow \quad \frac{3m-1}{6m(m+3)} \div \frac{4(3m-1)}{m+3} = \frac{3m-1}{6m(m+3)} \times \frac{m+3}{4(3m-1)}$$

We can cancel out the common terms $(3m-1)$ and $(m+3)$.

$$\Rightarrow \quad \frac{3m-1}{6m(m+3)} \times \frac{m+3}{4(3m-1)} = \frac{1}{6m} \times \frac{1}{4} = \frac{1}{24m} \quad \ldots\ldots\ldots (2)$$

Now we can add the simplified terms (1) and (2).

$$\Rightarrow \quad \frac{1}{12m} + \frac{1}{24m} = \frac{2}{24m} + \frac{1}{24m} = \frac{3}{24m} = \frac{1}{8m}$$

(xii) $\dfrac{1-p^2}{1+q}\times\dfrac{1-q^2}{p+p^2}\times\left(1+\dfrac{p}{1-p}\right)$

Solution:

Let's simplify the given expression:

$$\Rightarrow \quad \frac{1-p^2}{1+q}\times\frac{1-q^2}{p+p^2}\times\left(1+\frac{p}{1-p}\right)$$

First, factor the terms where possible:

$$\Rightarrow \quad 1-p^2=(1-p)(1+p) \qquad \Rightarrow \quad 1-q^2=(1-q)(1+q)$$

$$\Rightarrow \quad p+p^2=p(1+p)$$

$$\Rightarrow \quad 1+\frac{p}{1-p}=\frac{1-p}{1-p}+\frac{p}{1-p}=\frac{1-p+p}{1-p}=\frac{1}{1-p}$$

Substitute the factored terms into the expression:

$$=\frac{(1-p)(1+p)}{1+q}\times\frac{(1-q)(1+q)}{p(1+p)}\times\frac{1}{1-p}$$

Now, cancel common terms:

$$=\frac{1-q}{p}$$

(xiii) $\left[1-\dfrac{x}{1+\dfrac{x}{1-x}}\right]\div(1+x^3)$

Solution:

We are asked to simplify the expression:

$$\Rightarrow \quad \left[1-\frac{x}{1+\frac{x}{1-x}}\right]\div(1+x^3) \qquad \ldots\ldots\ldots(1)$$

First, let's simplify the expression inside the brackets:

$$1+\frac{x}{1-x}=\frac{1-x}{1-x}+\frac{x}{1-x}=\frac{1-x+x}{1-x}=\frac{1}{1-x}$$

Now we have: $1-\dfrac{x}{\frac{1}{1-x}}=1-x(1-x)=1-x+x^2$

So the expression (1) becomes: $(1-x+x^2)\div(1+x^3)=\dfrac{1-x+x^2}{1+x^3}$

Recall the sum of cubes factorization: $1+x^3=(1+x)(1-x+x^2)$

Therefore,

$$\Rightarrow \quad \frac{1-x+x^2}{(1+x)(1-x+x^2)}=\frac{1-x+x^2}{(1+x)(1-x+x^2)}=\frac{1}{1+x}$$

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