Exercise 5.1

Q1. Reduced the following expressions to their lowest form.

(i) $\frac{15ax^3y^2}{25a^2xy^6}$

Solution: $\frac{15ax^3y^2}{25a^2xy^6} = \frac{3}{5} \times a^{1-2} \times x^{3-1} \times y^{2-6}$

$= \frac{3}{5} \times a^{-1} \times x^2 \times y^{-4} = \frac{3x^2}{5ay^4}$

(ii) $\frac{38\,k^2p^3m^4}{57k^3pm^2}$

Solution: $\frac{38\,k^2p^3m^4}{57k^3pm^2} = \frac{38}{57} \times k^{2-3} \times p^{3-1} \times m^{4-2}$

$= \frac{2}{3} \times k^{-1} \times p^2 \times m^2 = \frac{2p^2m^2}{3k}$

(iii) $\frac{mn^4pq}{m^2n^3p^4}$

Solution: $\frac{mn^4pq}{m^2n^3p^4} = m^{1-2} \times n^{4-3} \times p^{1-4} \times q$

$= m^{-1} \times n^1 \times p^{-3} \times q = \frac{nq}{mp^3}$

(iv) $\frac{3abc}{15a^2b^2c}$

Solution: $\frac{3abc}{15a^2b^2c} = \frac{3}{15} \times a^{1-2} \times b^{1-2} \times c^{1-1} = \frac{1}{5} \times a^{-1} \times b^{-1} \times c^0$

$= \frac{1}{5} \times a^{-1} \times b^{-1} \times 1 = \frac{1}{5ab}$

(v) $\frac{46l^3m^4n^5}{69l^2m^3n^4}$

Solution: $\frac{46l^3m^4n^5}{69l^2m^3n^4} = \frac{46}{69} \times l^{3-2} \times m^{4-3} \times n^{5-4}$

$= \frac{2}{3} \times l^1 \times m^1 \times n^1 = \frac{2lmn}{3}$

(vi) $\frac{x-3}{3-x}$

Solution: $\frac{x-3}{3-x} = \frac{x-3}{-(x-3)} = -1$

(vii) $\frac{x^2-81}{x+9}$

Solution: $\frac{x^2-81}{x+9} = \frac{(x-9)(x+9)}{x+9} = x-9$

(viii) $\frac{(r+3)(r+4)}{r^2-16}$

Solution: $\frac{(r+3)(r+4)}{r^2-16} = \frac{(r+3)(r+4)}{(r-4)(r+4)} = \frac{r+3}{r-4}$

Q2. Evaluate the following expressions for the given value of each variable.

(i) $3(r^2 - s^2)$, if $r = 2$, $s = -1$.

Solution: $3(r^2 - s^2) = 3[(2)^2 - (-1)^2] = 3(4-1) = 3(3) = 9$

(ii) $\frac{1}{2}mv^2$ at $m = 18.75$ and $v = 5.6$

Solution: $\frac{1}{2}mv^2$; $m = 18.75$ and $v = 5.6$

$\frac{1}{2}mv^2 = \frac{1}{2}(18.75)(5.6)^2 = \frac{1}{2}(18.75)(31.36) = \frac{1}{2} \times 588 = 294$

(iii) $\sqrt{2gs}$ when $g = 32.2$ and $s = 144.9$

Solution: $\sqrt{2gs} = \sqrt{2 \times 32.2 \times 144.9} = \sqrt{9321.56} = \sqrt{9321.56} = 96.6$

(iv) $3x - y + \frac{1}{z}$ if $x = \frac{-1}{2}$, $y = 3$, $z = \frac{-1}{3}$

Solution: $3x - y + \frac{1}{z} = 3\left(\frac{-1}{2}\right) - 3 + \frac{1}{\left(\frac{-1}{3}\right)} = \frac{-3}{2} - 3 + (-3)$

$= \frac{-3}{2} - 3 - 3 = \frac{-3}{2} - 6 = \frac{-3}{2} - \frac{12}{2} = \frac{-3-12}{2} = \frac{-15}{2} = -7.5$

(v) $0.1d^2 + 0.01d + 1$ if $d = -0.2$

Solution: $0.1d^2 + 0.01d + 1 = 0.1(-0.2)^2 + 0.01(-0.2) + 1$

$= 0.1(0.04) + 0.01(-0.2) + 1$

$= 0.004 - 0.002 + 1 = 0.002 + 1 = 1.002$

(vi) $\frac{4}{7}b^3 - 3\frac{1}{2}b^2 + b - 3$ if $b = \frac{1}{2}$

Solution: $\frac{4}{7}b^3 - 3\frac{1}{2}b^2 + b - 3 = \frac{4}{7}b^3 - \frac{7}{2}b^2 + b - 3$

$= \frac{4}{7}\left(\frac{1}{2}\right)^3 - \frac{7}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{2} - 3 = \frac{4}{7}\left(\frac{1}{8}\right) - \frac{7}{2}\left(\frac{1}{4}\right) + \frac{1}{2} - 3$

$= \frac{1}{14} - \frac{7}{8} + \frac{1}{2} - 3$

The least common multiple of 14, 8 and 2 is 56.

So, we rewrite the fractions with a denominator of 56.

$\Rightarrow \frac{4}{56} - \frac{49}{56} + \frac{28}{56} - \frac{168}{56} = \frac{4 - 49 + 28 - 168}{56} = \frac{-185}{56} = -3\frac{17}{56}$

Q3. If nth triangular number is represented by $T(n) = \frac{n(n+1)}{2}$, then find 100^{th} triangular number.

Solution:

$\Rightarrow \quad T(n) = \frac{n(n+1)}{2}$ with $n = 100$.

$\Rightarrow \quad T(100) = \frac{100(100+1)}{2} = \frac{100 \times 101}{2} = \frac{10100}{2} = 5050$

So, the 100^{th} triangular number is 5050.

Q4. If $P(x) = x^2 + 2x - 15$, $D(x) = x - 3$ and $Q(x) = x + 5$, show that $\frac{P(2)}{Q(2)} = D(2)$.

Solution:

Given: $P(x) = x^2 + 2x - 15$; $D(x) = x - 3$; $Q(x) = x + 5$

Evaluate $P(2)$.

$\Rightarrow \quad P(2) = (2)^2 + 2(2) - 15 = 4 + 4 - 15 = 8 - 15 = -7$

Evaluate $Q(2)$. $\Rightarrow \quad Q(2) = (2) + 5 = 7$

Evaluate $D(2)$. $\Rightarrow \quad D(2) = (2) - 3 = -1$

Now, we compute $\frac{P(2)}{Q(2)}$. $\Rightarrow \quad \frac{P(2)}{Q(2)} = \frac{-7}{7} = -1$

Since, $\frac{P(2)}{Q(2)} = -1$ and $D(2) = -1$, we have: $\frac{P(2)}{Q(2)} = D(2)$

Therefore, it is shown that $\frac{P(2)}{Q(2)} = D(2)$.

Q5. If $g(x)=\frac{1}{2x^3}+\frac{x}{2}+2$, find $g\left(-\frac{1}{3}\right)$.

Solution:

$\Rightarrow \quad g(x)=\frac{1}{2x^3}+\frac{x}{2}+2$

$\Rightarrow \quad g\left(-\frac{1}{3}\right)=\frac{1}{2\left(-\frac{1}{3}\right)^3}+\frac{-\frac{1}{3}}{2}+2 \qquad \Rightarrow \quad g\left(-\frac{1}{3}\right)=\frac{1}{2\left(-\frac{1}{27}\right)}-\frac{1}{6}+2$

$\Rightarrow \quad g\left(-\frac{1}{3}\right)=\frac{1}{-\frac{2}{27}}-\frac{1}{6}+2 \qquad \Rightarrow \quad g\left(-\frac{1}{3}\right)=-\frac{27}{2}-\frac{1}{6}+2$

$\Rightarrow \quad g\left(-\frac{1}{3}\right)=\frac{-81-1+12}{6} \qquad \Rightarrow \quad g\left(-\frac{1}{3}\right)=\frac{-82+12}{6}$

$\Rightarrow \quad g\left(-\frac{1}{3}\right)=\frac{-70}{6} \qquad \Rightarrow \quad g\left(-\frac{1}{3}\right)=-\frac{35}{3}=-11\frac{2}{3}$

Q6. **The volume of a basketball (sphere) is approximately 38808 cm^3.**

The radius 'r' of the ball is given by $r=\sqrt[3]{\frac{3v}{4\pi}}$, where v is its volume.

Determine the radius of that ball. (take $\pi=\frac{22}{7}$)

Solution: $\quad r=\sqrt[3]{\frac{3v}{4\pi}}$

$\Rightarrow \quad v=38808\ \text{cm}^3$ and $\pi=\frac{22}{7}$

$\Rightarrow \quad r=\sqrt[3]{\frac{3\times 38808}{4\times\frac{22}{7}}}$

First, simplify the expression inside the cube root:

$\Rightarrow \quad r=\sqrt[3]{\frac{3\times 38808\times 7}{4\times 22}}=\sqrt[3]{\frac{3\times 38808\times 7}{88}}=\sqrt[3]{\frac{814968}{88}} \qquad \Rightarrow \quad r=\sqrt[3]{9261}$

Now, find the cube root of **9261.**

$\Rightarrow \quad r=\sqrt[3]{9261}=21\ \text{cm}$

So, the radius of the basketball is **21 cm.**

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