Exercise 4.4

Q1. Describe the steps you would take to graph the system of inequalities shown.

$$x - y < 6 \;\; \text{.........} \;\; \textbf{(Inequality I)}$$
$$y \geq 3 \;\; \text{.........} \;\; \textbf{(Inequality II)}$$

Solution:

Steps to graph the given system of inequalities:

(i) Inequality I:

$\Rightarrow$ $x - y < 6$

- **Step 1:**
  Rewrite the inequality as an equation: $x - y = 6$
- **Step 2:**
  Solve for y to get the slope-intercept form: $y = x - 6$
- **Step 3:**
  Graph the line $y = x - 6$. Since the inequality is "<" (less than), use a dashed line to indicate that the points on the line are not included in the solution.
- **Step 4:**
  Determine the region to shade. Since the inequality is $y > x - 6$, shade the region above the dashed line.

(ii) Inequality II:

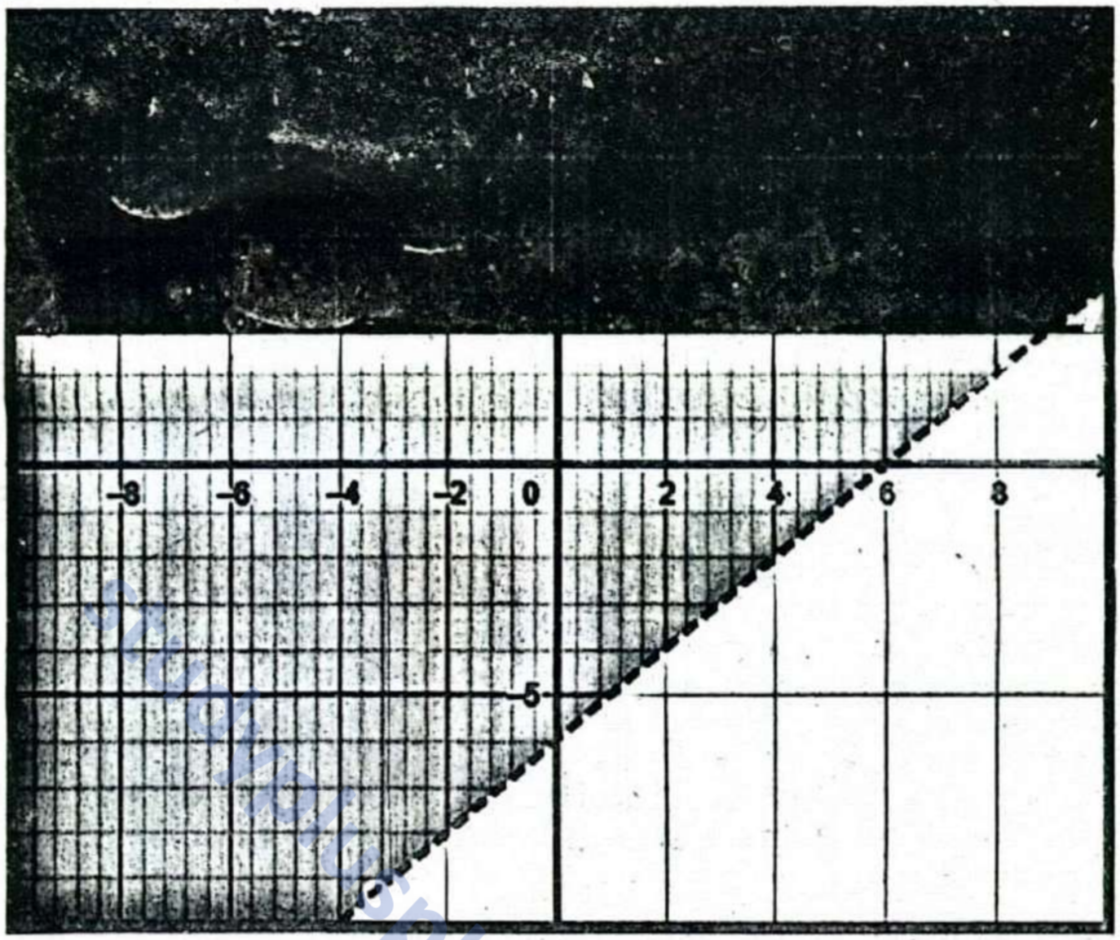
$\Rightarrow$ $y \geq 3$

- **Step 1:**
  Rewrite the inequality as an equation: $y = 3$
- **Step 2:**
  Graph the horizontal line $y = 3$. Since the inequality is "≥" (greater than or equal to), use a solid line to indicate that the points on the line are included in the solution.
- **Step 3:**
  Determine the region to shade. Since the inequality is $y \geq 3$, shade the region above the solid line.

***(iii)* Solution:**

The solution to the system of inequalities is the region where the shaded areas of both inequalities overlap. This region represents all the points (x, y) that satisfy both inequalities simultaneously.

- **Graph of the solution:**



Q2. Tell whether the ordered pair is a solution of the system of inequalities.

a. $(3,1)$, $(0,0)$, $(-3,1)$, $(4,1)$; $x+y<1$, $x-y<2$

Solution:

We will determine whether each ordered pair is a solution to the system of inequalities:

$\Rightarrow \begin{cases} x+y<1 \\ x-y<2 \end{cases}$

(i) Consider $(3,1)$:

- $x+y<1 \quad \Rightarrow \quad 3+1<1 \quad \Rightarrow \quad 4<1$ (False)
- $x-y<2 \quad \Rightarrow \quad 3-1<2 \quad \Rightarrow \quad 2<2$ (False)

Since both inequalities are false, $(3,1)$ is not a solution.

(ii) Consider $(0,0)$:

- $x+y<1 \quad \Rightarrow \quad 0+0<1 \quad \Rightarrow \quad 0<1$ (True)
- $x-y<2 \quad \Rightarrow \quad 0-0<2 \quad \Rightarrow \quad 0<2$ (True)

Since both inequalities are true, $(0,0)$ is a solution.

(iii) *Consider* $(-3,1)$:

- $x+y<1 \quad \Rightarrow \quad -3+1<1 \quad \Rightarrow \quad -2<1$ (True)
- $x-y<2 \quad \Rightarrow \quad -3-1<2 \quad \Rightarrow \quad -4<2$ (True)

Since both inequalities are true, $(-3,1)$ is a solution.

(iv) Consider $(4, 1)$:

- $x + y < 1 \quad \Rightarrow \quad 4 + 1 < 1 \quad \Rightarrow \quad 5 < 1$ (False)
- $x - y < 2 \quad \Rightarrow \quad 4 - 1 < 2 \quad \Rightarrow \quad 3 < 2$ (False)

Since both inequalities are false, $(4, 1)$ is not a solution.

In summary:

(i) $(3, 1)$: Not a solution **(ii)** $(0, 0)$: is a solution
(iii) $(-3, 1)$: is a solution **(iv)** $(4, 1)$: Not a solution

$\Rightarrow$ $(0, 0)$ and $(-3, 1)$ are solutions.

b. $(1, -1)$, $(4, 1)$, $(2, 0)$, $(3, 2)$; $2x - y \leq 5$, $x + 2y > 2$

Solution:

We will determine whether each ordered pair is a solution to the system of inequalities:

$$\Rightarrow \begin{cases} 2x - y \leq 5 \\ x + 2y > 2 \end{cases}$$

(i) Consider $(1, -1)$:

- $2x - y \leq 5 \quad \Rightarrow \quad 2(1) - (-1) \leq 5$

$\Rightarrow$ $2 + 1 \leq 5 \quad \Rightarrow \quad 3 \leq 5$ (True)

- $x + 2y > 2 \quad \Rightarrow \quad 1 + 2(-1) > 2$

$\Rightarrow$ $1 - 2 > 2 \quad \Rightarrow \quad -1 > 2$ (False)

Since one inequality is false, $(1, -1)$ is not a solution.

(ii) Consider $(4, 1)$:

- $2x - y \leq 5 \quad \Rightarrow \quad 2(4) - 1 \leq 5$

$\Rightarrow$ $8 - 1 \leq 5 \quad \Rightarrow \quad 7 \leq 5$ (False)

- $x + 2y > 2 \quad \Rightarrow \quad 4 + 2(1) > 2$

$\Rightarrow$ $4 + 2 > 2 \quad \Rightarrow \quad 6 > 2$ (True)

Since one inequality is false, $(4, 1)$ is not a solution.

(iii) Consider $(2, 0)$:

- $2x - y \leq 5 \quad \Rightarrow \quad 2(2) - 0 \leq 5$

$\Rightarrow$ $4 - 0 \leq 5 \quad \Rightarrow \quad 4 \leq 5$ (True)

- $x + 2y > 2 \quad \Rightarrow \quad 2 + 2(0) > 2$

$\Rightarrow$ $2 + 0 > 2 \quad \Rightarrow \quad 2 > 2$ (False)

Since one inequality is false, $(2, 0)$ is not a solution.

(iv) Consider $(3, 2)$:

- $2x - y \leq 5 \quad \Rightarrow \quad 2(3) - 2 \leq 5$

$\Rightarrow$ $6 - 2 \leq 5 \quad \Rightarrow \quad 4 \leq 5$ (True)

- $x + 2y > 2 \quad \Rightarrow \quad 3 + 2(2) > 2$

$\Rightarrow$ $3 + 4 > 2 \quad \Rightarrow \quad 7 > 2$ (True)

Since both inequalities are true, $(3, 2)$ is a solution.

In summary:

(i) $(1, -1)$: Not a solution **(ii)** $(4, 1)$: Not a solution
(iii) $(2, 0)$: Not a solution **(iv)** $(3, 2)$: is a solution

$\Rightarrow$ $(3, 2)$ is solution.

Q3. **Graph the system of inequalities.**

a. $-x + y < 0$

$x + y > 3$

Solution:

$\Rightarrow \quad \begin{cases} -x + y < 0 \\ x + y > 3 \end{cases}$

First, we will rewrite each inequality in slope-intercept form:

(i) **Inequality 1:**

$\Rightarrow \quad -x + y < 0$

$\Rightarrow \quad y < x$

(ii) **Inequality 2:**

$\Rightarrow \quad x + y > 3$

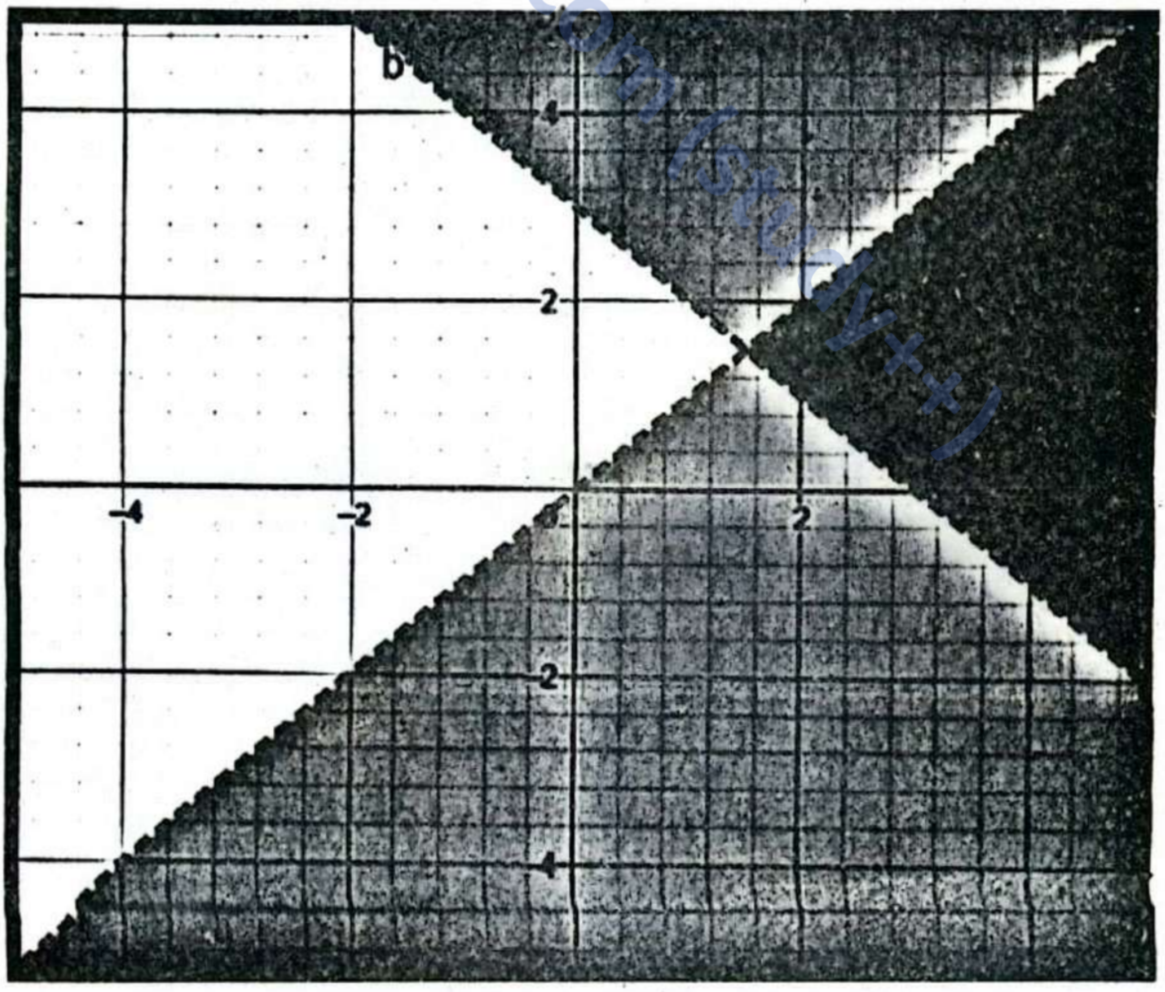
$\Rightarrow \quad y > -x + 3$

Now, we will graph these inequalities.

- For $y < x$, we will graph the line $y = x$ as a dashed line (since it's strictly less than) and shade the region below the line.
- For $y > -x + 3$, we will graph the line $y = -x + 3$ as a dashed line (since it's strictly greater than) and shade the region above the line.

The solution to the system of inequalities is the region where the shaded areas of both inequalities overlap.

- **Graph of the solution:**



b. $x - y < 0$

$x + y < 1$

Solution:

The given inequalities are:

$x - y < 0$

$x + y < 1$

Let's rewrite the first inequality:

$\Rightarrow \quad x - y < 0$

$\Rightarrow \quad x < y$

$\Rightarrow \quad y > x$

Now, let's rewrite the second inequality:

$\Rightarrow \quad x + y < 1$

$\Rightarrow \quad y < -x + 1$

Now we have the two inequalities:

$\Rightarrow \quad y > x$

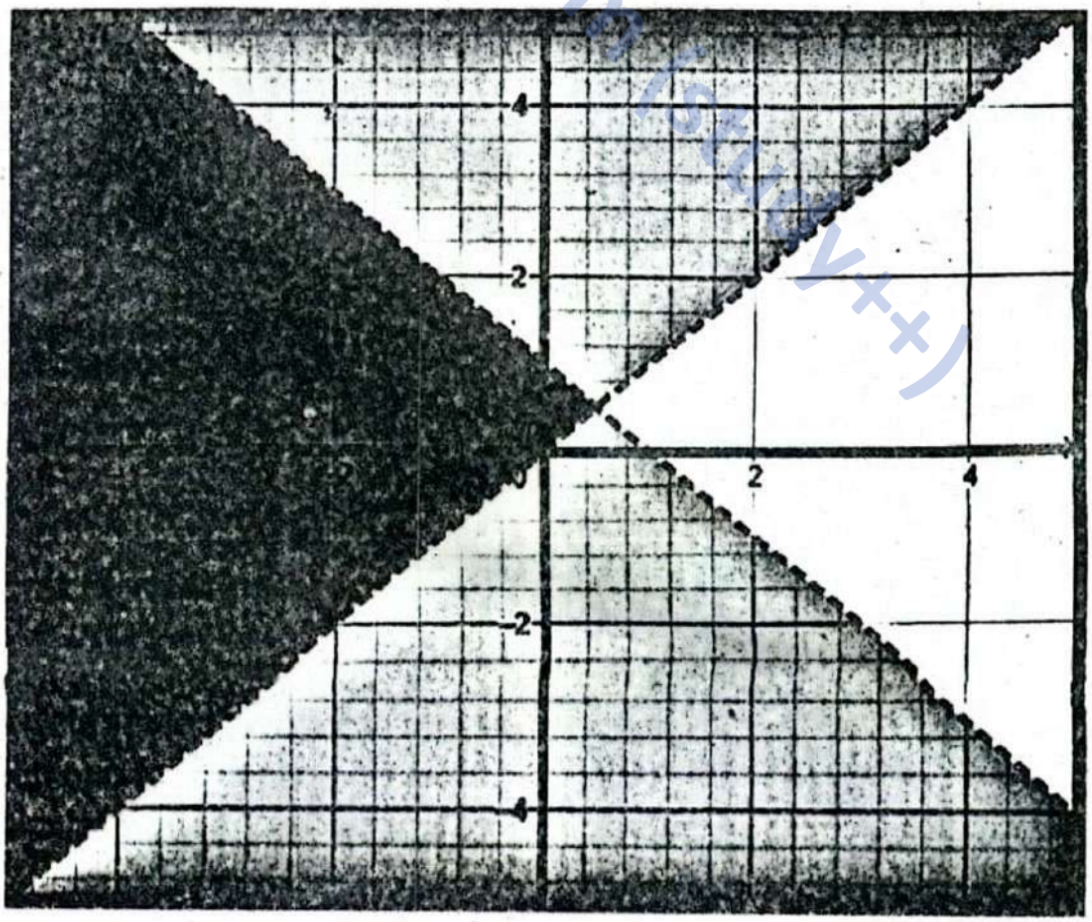
$\Rightarrow \quad y < -x + 1$

The first inequality, $y > x$, represents the region above the line $y = x$. Since it's a strict inequality (>), we use a dashed line to indicate that the line itself is not included in the solution.

The second inequality, $y < -x + 1$, represents the region below the line $y = -x + 1$. Since it's also a strict inequality (<), we use a dashed line to indicate that the line itself is not included in the solution.

The solution to the system of inequalities is the region where the two shaded regions overlap.

- **Graph of the solution:**



c. $y - 2x \geq 3$

$y - 2x \leq 3$

Solution:

To graph the system of inequalities, we first need to rewrite each inequality in slope-intercept form $(y = mx + b)$.

The given inequalities are:

$\Rightarrow \quad y - 2x \geq 3y - 2x \leq 3$

Let's rewrite the first inequality:

$\Rightarrow \quad y - 2x \geq 3y \geq 2x + 3$

Now, let's rewrite the second inequality:

$\Rightarrow \quad y - 2x \leq 3y \leq 2x + 3$

Now we have the two inequalities:

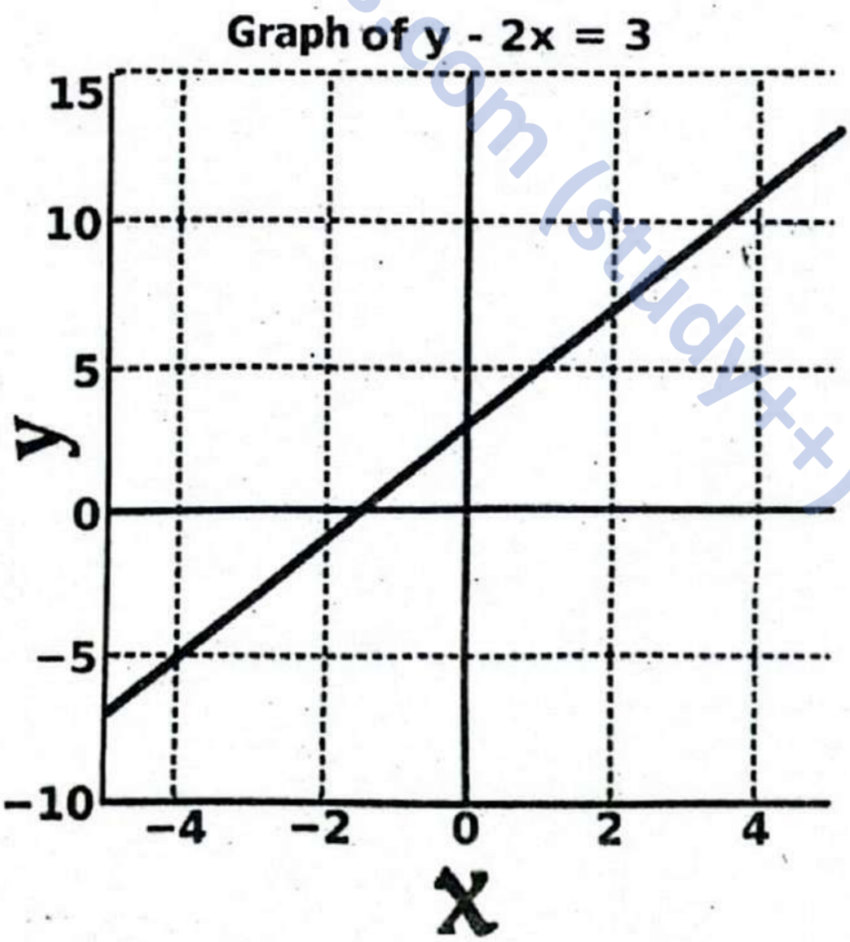
$\Rightarrow \quad y \geq 2x + 3y \leq 2x + 3$

Notice that both inequalities represent the same line, $y = 2x + 3$. The first inequality includes the region above the line, and the second inequality includes the region below the line. Since both inequalities also include the line itself (due to the $\geq$ and $\leq$ symbols), the solution to the system is the line $y = 2x + 3$.

Graphically, we would draw a solid line at $y = 2x + 3$. There is no region to shade because the solution is only the line.

Both lines are parallel and superpose each other.

- **<u>Graph of the solution:</u>**



d. $y \leq -2$

$x \geq 2$

Solution:

To graph the system of inequalities:

⇒ $y \leq -2x \geq 2$

The inequality $y \leq -2$ represents the region on or below the horizontal line $y = -2$.

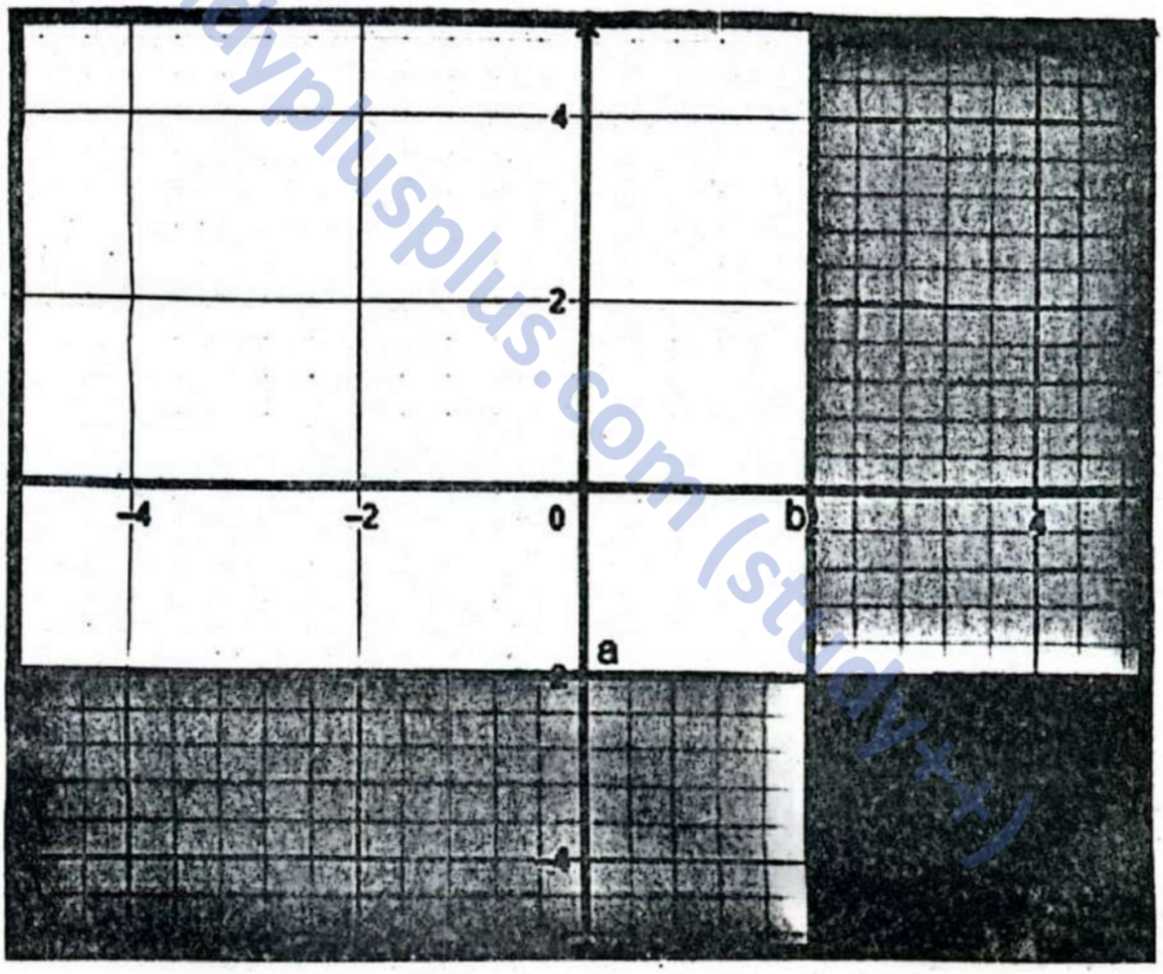
Since it includes the equals sign, we draw a solid line at $y = -2$.

The inequality $x \geq 2$ represents the region on or to the right of the vertical line $x = 2$.

Since it includes the equals sign, we draw a solid line at $x = 2$.

The solution to the system of inequalities is the region where the two shaded regions overlap, which is the region in the fourth quadrant where $x \geq 2$ and $y \leq -2$.

- **Graph of the solution:**



e. $x > -2$

$2x + y < 3$

Solution:

To graph the system of inequalities, we first need to rewrite each inequality in terms of y where applicable.

The given inequalities are:

⇒ $x > -22x + y < 3$

The first inequality, $x > -2$, is already in a suitable form. It represents the region to the right of the vertical line $x = -2$. Since it's a strict inequality ($>$), we use a dashed line to indicate that the line itself is not included in the solution.

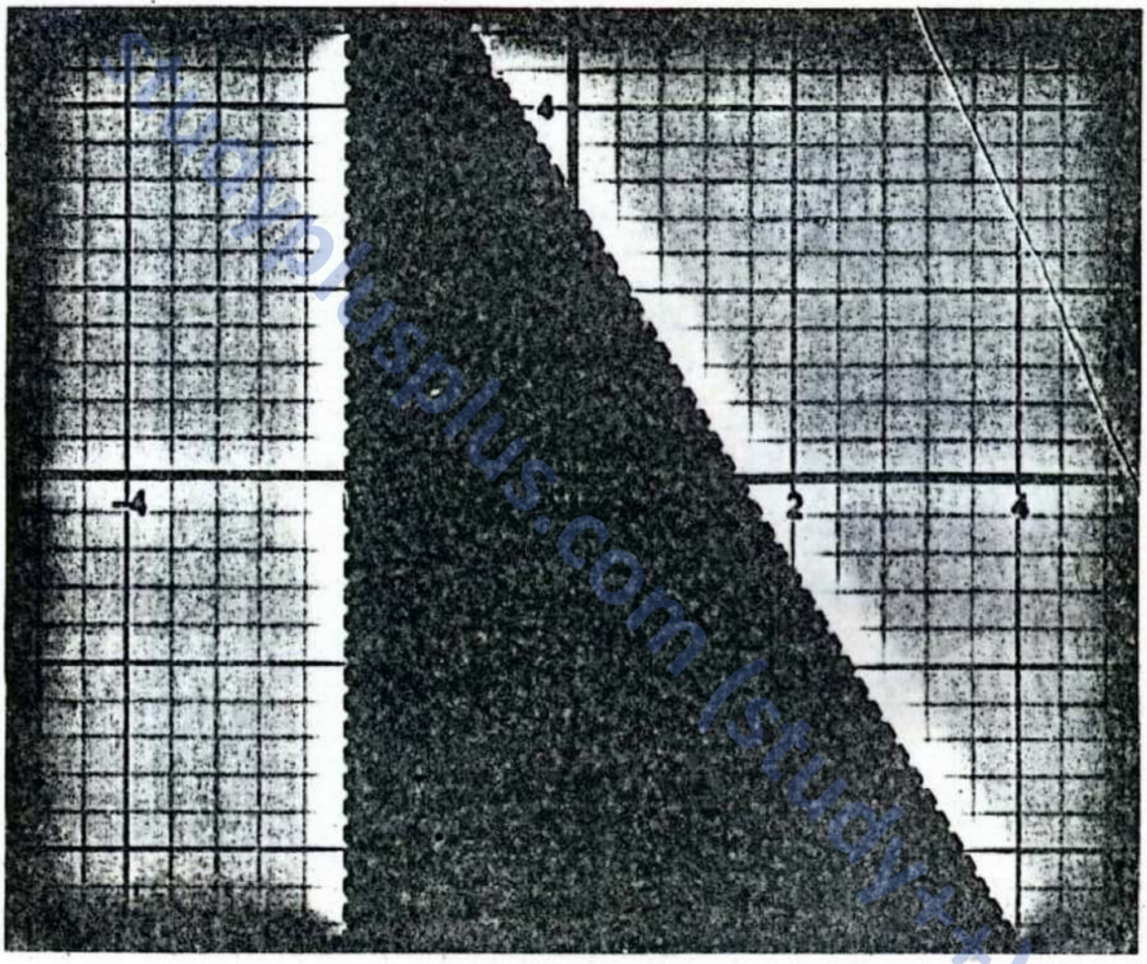
Now, let's rewrite the second inequality:

$\Rightarrow \quad 2x + y < 3y < -2x + 3$

The second inequality, $y < -2x + 3$, represents the region below the line $y = -2x + 3$. Since it's a strict inequality ($<$), we use a dashed line to indicate that the line itself is not included in the solution.

The solution to the system of inequalities is the region where the two shaded regions overlap.

- **Graph of the solution:**



f. $x + y \leq 3$

$x - y \leq 4$

Solution:

$x + y \leq 3$

$x - y \leq 4$

First, we rewrite each inequality in slope-intercept form:

$\Rightarrow \quad x + y \leq 3$

$\Rightarrow \quad y \leq -x + 3$

And

$\Rightarrow \quad x - y \leq 4$

$\Rightarrow \quad -y \leq -x + 4$

$\Rightarrow \quad y \geq x - 4$

So we have the system:

$\Rightarrow \quad y \leq -x + 3$

$\Rightarrow \quad y \geq x - 4$

The first inequality, $y \leq -x + 3$, has a boundary line of $y = -x + 3$. The y-intercept is 3 and the slope is -1. We draw a solid line since the inequality includes equality, and we shade below the line.

The second inequality, $y \geq x - 4$, has a boundary line of $y = x - 4$. The y-intercept is -4 and the slope is 1. We draw a solid line since the inequality includes equality, and we shade above the line.

The solution set is the intersection of these two shaded regions.

Let's find the intersection point of the two lines:

$\Rightarrow \quad -x + 3 = x - 4$

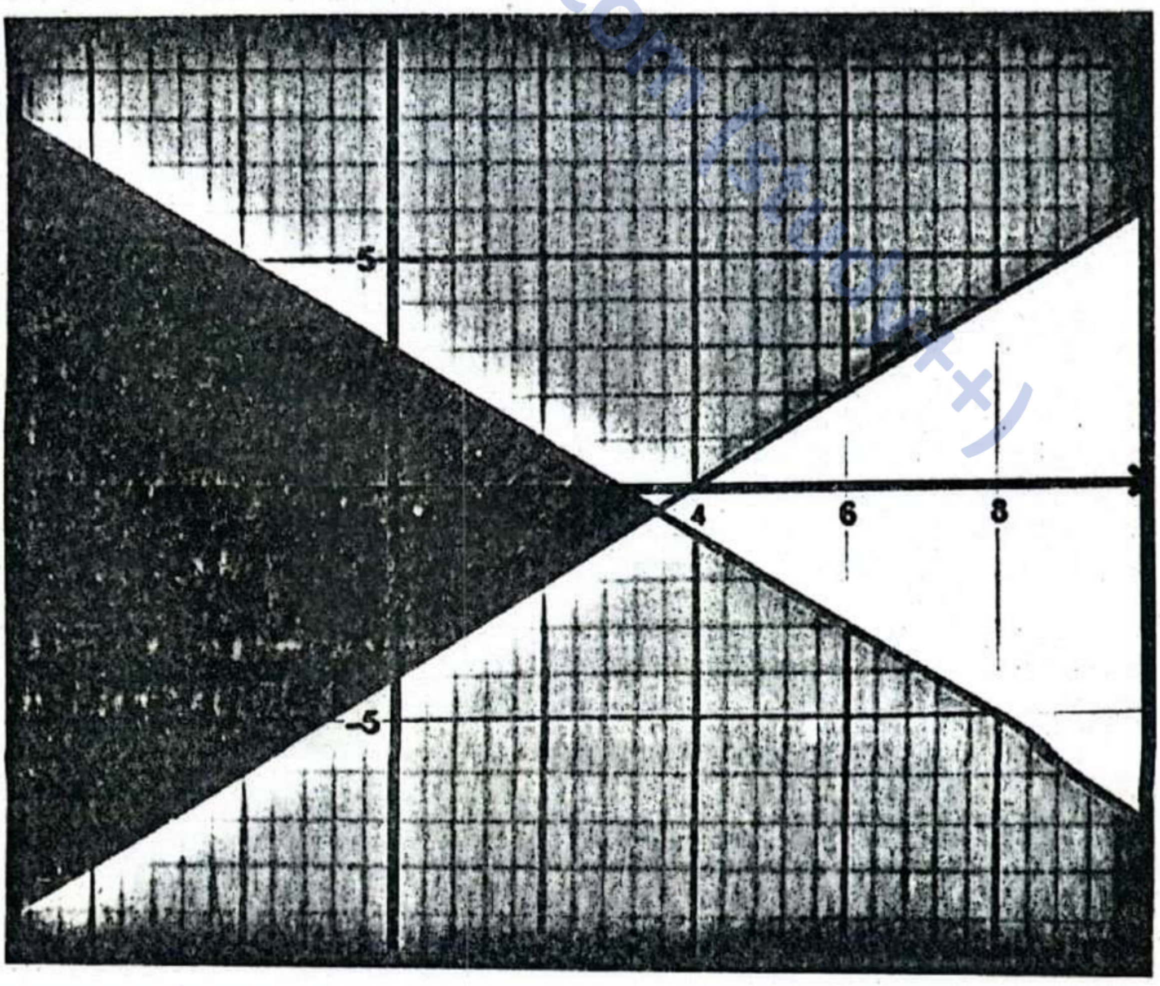
$\Rightarrow \quad 7 = 2x$

$\Rightarrow \quad x = \frac{7}{2} = 3.5$

When $x = \frac{7}{2}$, $y = \frac{7}{2} - 4 = \frac{7}{2} - \frac{8}{2} = -\frac{1}{2} = -0.5$

The intersection point is $\left(\frac{7}{2}, -\frac{1}{2}\right) = (3.5, -0.5)$.

- **<u>Graph of the solution:</u>**



g. $y \ge 2x = 1$
$y < -x + 4$

Solution:

$y \ge 2x + 1$
$y < -x + 4$

The first inequality, $y \ge 2x + 1$, has a boundary line of $y = 2x + 1$. The y-intercept is 1 and the slope is 2. We draw a solid line since the inequality includes equality, and we shade above the line.

The second inequality, $y < -x + 4$, has a boundary line of $y = -x + 4$. The y-intercept is 4 and the slope is -1. We draw a dashed line since the inequality does not include equality, and we shade below the line.

The solution set is the intersection of these two shaded regions.

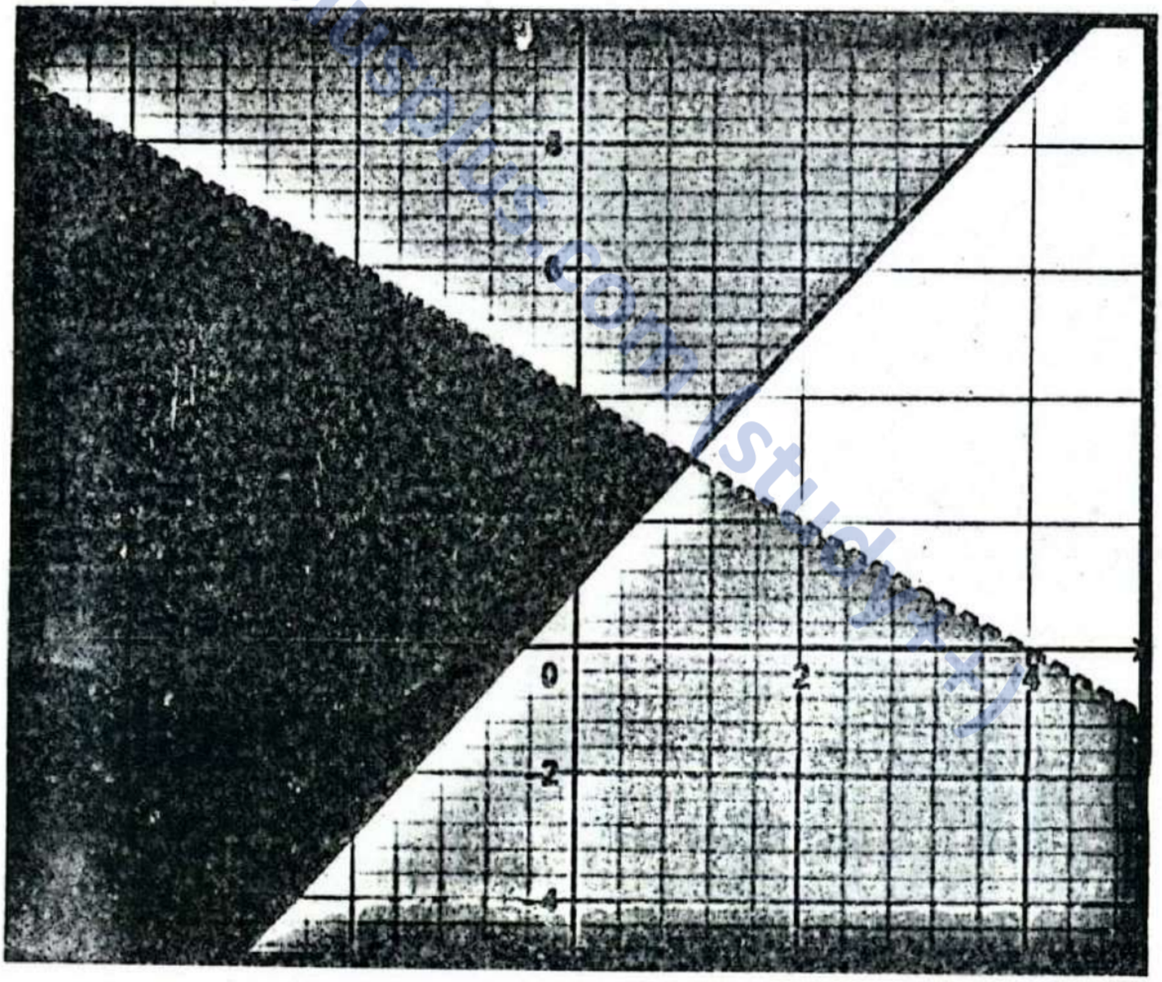
Let's find the intersection point of the two lines:

$\Rightarrow \quad 2x + 1 = -x + 4 \quad \Rightarrow \quad 3x = 3 \quad \Rightarrow \quad x = 1$

When $x = 1$, $y = 2(1) + 1 = 3$.

The intersection point is $(1, 3)$.

- **<u>Graph of the solution:</u>**



h. $x < 8$
$x - 4y \le -8$

Solution:

$$\begin{cases} x < 8 \\ x - 4y \le -8 \end{cases}$$

First, we can rewrite the second inequality to isolate y.

$\Rightarrow$ $x - 4y \leq -8 - 4y \leq -x - 84y \geq x + 8y \geq \frac{1}{4}x + 2$

So the system becomes:

$$\begin{cases} x < 8 \\ y \geq \frac{1}{4}x + 2 \end{cases}$$

Now we can graph these inequalities.

(i) $x < 8$:

This is a vertical line at $x = 8$. Since the inequality is strict ($<$), we draw a dashed line at $x = 8$. The region to the left of this line is the solution to this inequality.

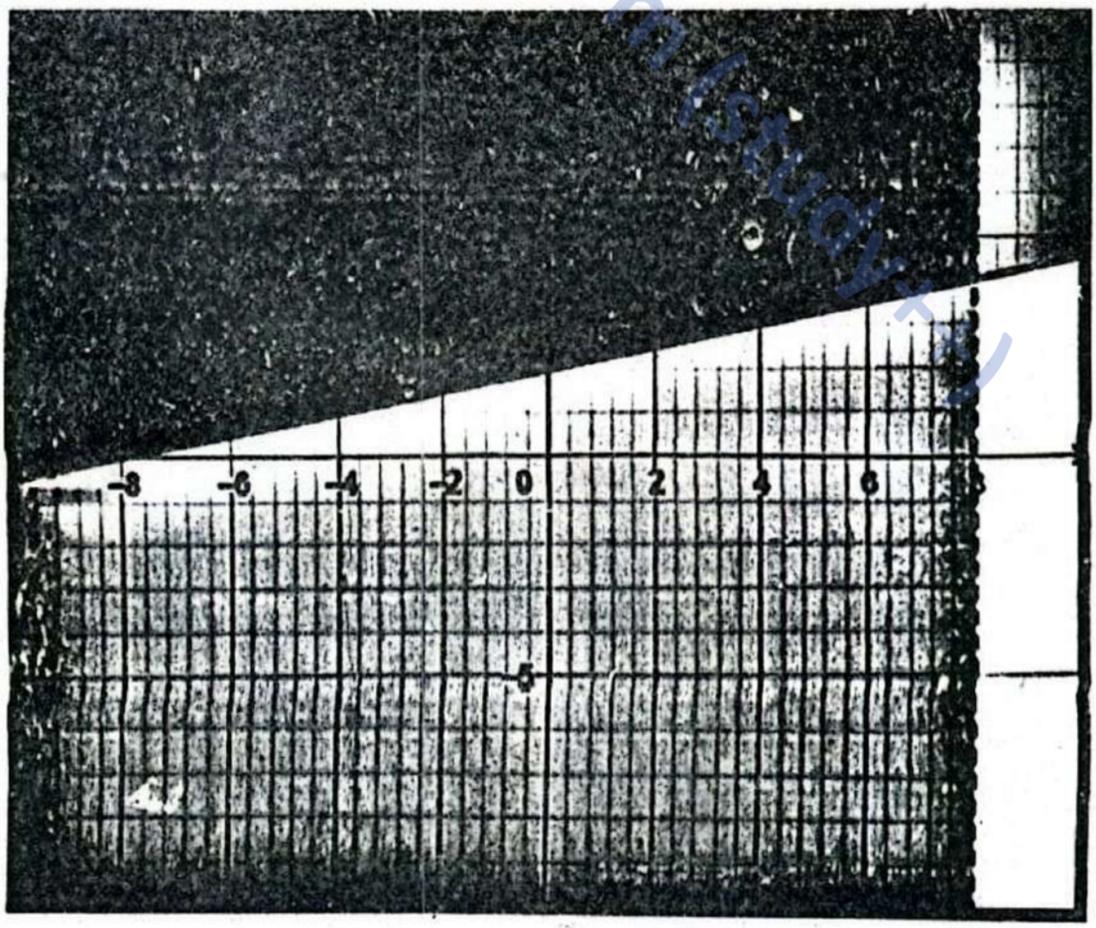
(II) $y \geq \frac{1}{4}x + 2$:

This is a linear inequality. The boundary line is $y = \frac{1}{4}x + 2$. The y-intercept is 2, and the slope is $\frac{1}{4}$.

Since the inequality is non-strict ($\geq$), we draw a solid line. The region above this line is the solution to this inequality.

The solution to the system is the intersection of these two regions. We need to find the region that is to the left of the dashed line $x = 8$ and above the solid line $y = \frac{1}{4}x + 2$.

- **Graph of the solution:**



Q4. Does the system of inequalities have any solution?

$$x - y > 5$$
$$x - y < 1$$

Solution:

Let's analyze the system of inequalities: $\begin{cases} x - y > 5 \\ x - y < 1 \end{cases}$

We can rewrite these inequalities as: $\begin{cases} y < x - 5 \\ y > x - 1 \end{cases}$

Now, let's consider what these inequalities mean graphically.

- $y < x - 5$ represents the region below the line $y = x - 5$.
- $y > x - 1$ represents the region above the line $y = x - 1$.

Notice that the lines $y = x - 5$ and $y = x - 1$ are parallel, since they both have a slope of 1. The line $y = x - 1$ is above the line $y = x - 5$.

The system of inequalities requires us to find a region where y is both less than $x - 5$ and greater than $x - 1$.

However, since $x - 1$ is always greater than $x - 5$, there is no value of y that can satisfy both inequalities simultaneously.

Therefore, the system of inequalities has no solution.

Alternatively, we can show this algebraically.

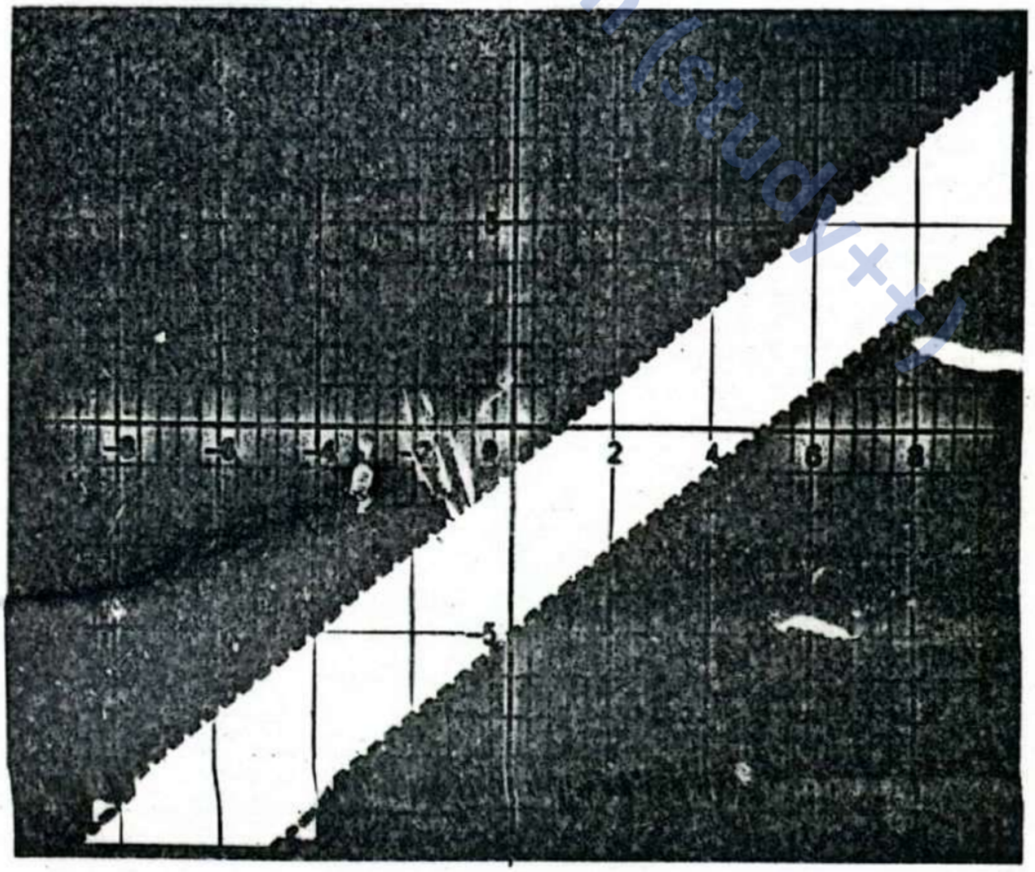
Suppose there is a solution (x, y).

Then we must have $x - y > 5$ and $x - y < 1$.

This implies that $5 < x - y < 1$, which is a contradiction, since 5 cannot be less than 1.

Therefore, there is no solution.

- <u>**Graph of the solution:**</u>



Q5. Open-Ended Question: Describe a real-world situation that can be modeled by a system of linear inequalities. Then write and graph the system of inequalities.

Solution:

- **Real-World Situation:**

  Suppose you are planning a party and have a budget of $\$200$ for food and drinks. You want to buy pizza and soda. Each pizza costs $\$15$, and each bottle of soda costs $\$2$. You also want to have at least 5 pizzas for your guests.

- **System of Inequalities:**

  Let x be the number of pizzas and y be the number of bottles of soda.
  We can write the following inequalities:

(i) Budget constraint:

The total cost of pizzas and soda must be less than or equal to $\$200$.

$$15x + 2y \leq 200$$

(ii) Minimum number of pizzas:

You want at least 5 pizzas:

$$x \geq 5$$

(iii) Non-negativity:

The number of pizzas and soda bottles cannot be negative:

$$x \geq 0$$
$$y \geq 0$$

- **Graphing the System:**

  We need to graph the inequalities:

(i) $15x + 2y \leq 200$:

We can rewrite this as $y \leq -\frac{15}{2}x + 100$.

The boundary line is $y = -\frac{15}{2}x + 100$. The region below this line satisfies the inequality.

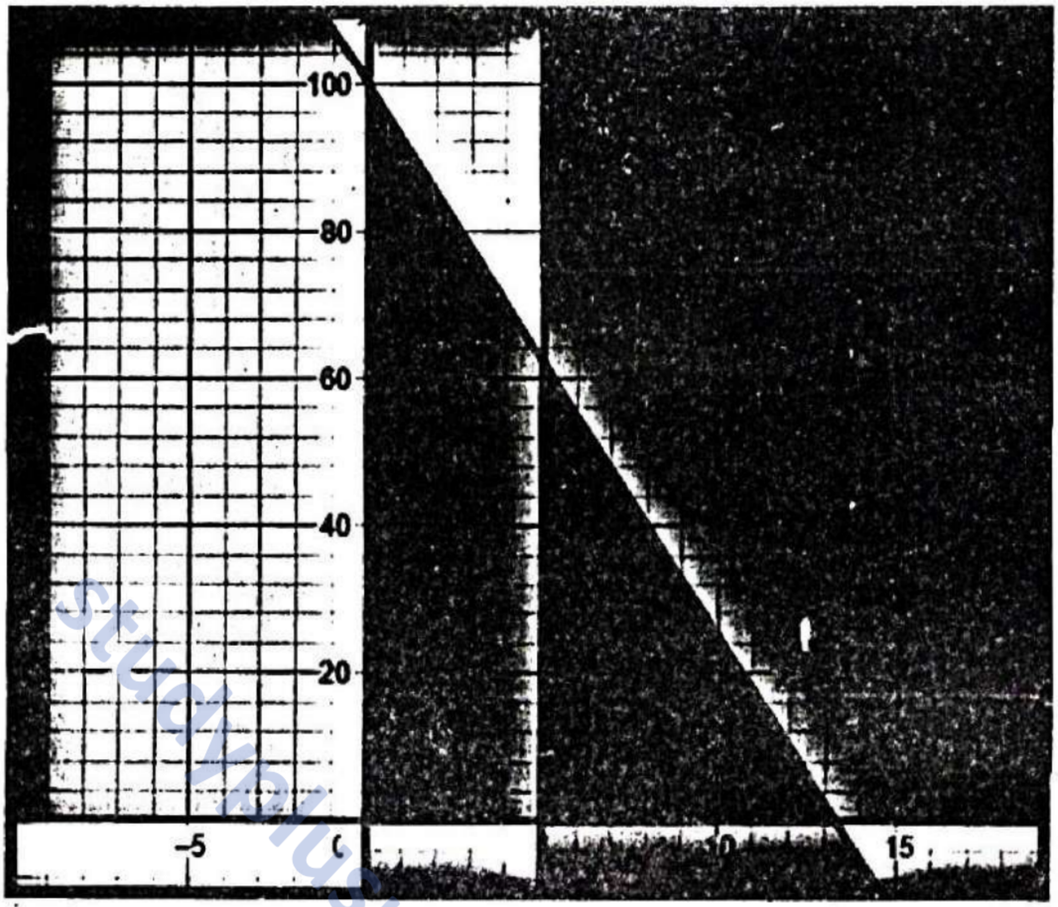
(ii) $x \geq 5$:

This is a vertical line at $x = 5$. The region to the right of this line satisfies the inequality.

(iii) $x \geq 0$ and $y \geq 0$:

These restrict the solution to the first quadrant.

- **Graph of the solution:**



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