Exercise 4.3

> **Solve the absolute valued equations (1-6).**

Q1. $8|x-3| = 88$

Solution: $8|x-3| = 88$

$\Rightarrow \quad |x-3| = \frac{88}{8} \quad \Rightarrow \quad |x-3| = 11$

This gives us two equations:

Equation 1: $x-3 = 11$; **Equation 2:** $x-3 = -11$

Solve each equation:

(i) **Equation 1:**

$\Rightarrow \quad x-3 = 11 \quad \Rightarrow \quad x = 11+3 \quad \Rightarrow \quad x = 14$

(ii) **Equation 2:**

$\Rightarrow \quad x-3 = -11 \quad \Rightarrow \quad x = -11+3 \quad \Rightarrow \quad x = -8$

$\therefore$ Solution set $= \{(-8, 14)\}$

Q2. $|2x+9| = 30$

Solution: $|2x+9| = 30$

This gives us two equations:

Equation 1: $2x+9 = 30$; **Equation 2:** $2x+9 = -30$

Solve each equation:

(i) **Equation 1:**

$\Rightarrow \quad 2x+9 = 30 \quad \Rightarrow \quad 2x = 30-9 \quad \Rightarrow \quad 2x = 21 \quad \Rightarrow \quad x = \frac{21}{2}$

(ii) **Equation 2:**

$\Rightarrow \quad 2x+9 = -30 \quad \Rightarrow \quad 2x = -30-9$

$\Rightarrow \quad 2x = -39 \quad \Rightarrow \quad x = -\frac{39}{2}$

$\therefore$ Solution set $= \left\{\left(-\frac{39}{2}, \frac{21}{2}\right)\right\}$

Q3. $|x-3| = -26$

Solution: $|x-3| = -26$

Since the absolute value of non-zero integer is always positive.

Therefore absolute value cannot be negative, there is **no solution.**

Q4. $3|x+6| = 9x-6$

Solution: $3|x+6| = 9x-6$

Because of the absolute value, we need to consider two cases:

(i) **Case 1:**

The expression inside the absolute value is positive or zero:

$\Rightarrow \quad x + 6 = 3x - 2$

(ii) **Case 2:**

The expression inside the absolute value is negative:

$\Rightarrow \quad x + 6 = -(3x - 2)$

Solve each equation:

(i) **Case 1:**

$\Rightarrow \quad x + 6 = 3x - 2 \quad \Rightarrow \quad 6 = 2x - 2 \quad \Rightarrow \quad 8 = 2x \quad \Rightarrow \quad x = 4$

(ii) **Case 2:**

$\Rightarrow \quad x + 6 = -(3x - 2) \quad \Rightarrow \quad x + 6 = -3x + 2$

$\Rightarrow \quad 4x + 6 = 2 \quad \Rightarrow \quad 4x = -4 \quad \Rightarrow \quad x = -1$

Check for extraneous solutions:

We need to check if our solutions are valid by plugging them back into the original equation $3|x + 6| = 9 - 6$.

Check $x = 4$:

$\Rightarrow \quad 3|4 + 6| = 9(4) - 6 \quad \Rightarrow \quad 3|10| = 36 - 6 \quad \Rightarrow \quad 30 = 30$

This is true, so $x = 4$ is a valid solution.

Check $x = -1$:

$\Rightarrow \quad 3|-1 + 6| = 9(-1) - 6 \quad \Rightarrow \quad 3|5| = -9 - 6 \quad \Rightarrow \quad 15 \neq -15$

This is false, so $x = -1$ is an extraneous solution.

The only solution to the equation is $x = 4$.

$\therefore$ **Solution set** $= \{(4)\}$

Q8. $|x - 3| = 2x$

Solution: $|x - 3| = 2x$

Because of the absolute value, we need to consider two cases:

(i) **Case 1:**

The expression inside the absolute value is positive or zero:

$\Rightarrow \quad x - 3 = 2x$

(ii) **Case 2:**

The expression inside the absolute value is negative:

$\Rightarrow \quad x - 3 = -(2x)$

Solve each equation:

(i) **Case 1:**

$\Rightarrow \quad x - 3 = 2x \quad \Rightarrow \quad -3 = x \quad \Rightarrow \quad$ So, $x = -3$

(ii) **Case 2:**

$\Rightarrow \quad x - 3 = -(2x) \quad \Rightarrow \quad x - 3 = -2x$

$\Rightarrow \quad 3x - 3 = 0 \quad \Rightarrow \quad 3x = 3 \quad \Rightarrow \quad x = 1$

- **Check for extraneous solutions:**

We need to check if our solutions are valid by plugging them back into the original equation $|x - 3| = 2x$.

Check $x = -3$:

$\Rightarrow \quad |-3 - 3| = 2(-3) \quad \Rightarrow \quad |-6| = -6 \quad \Rightarrow \quad 6 \neq -6$

This is false, so $x = -3$ is an extraneous solution.

Check $x = 1$:

$\Rightarrow \quad |1 - 3| = 2(1) \quad \Rightarrow \quad |-2| = 2 \quad \Rightarrow \quad 2 = 2$

This is true, so $x = 1$ is a valid solution.

$\therefore$ **Solution set** $= \{(1)\}$

Q6. $\left|\frac{4 - 5x}{6}\right| = 7$

Solution: $\left|\frac{4 - 5x}{6}\right| = 7$

Because of the absolute value, we need to consider two cases:

(i) **Case 1:**

The expression inside the absolute value is positive or zero: $\frac{4 - 5x}{6} = 7$

(ii) **Case 2:**

The expression inside the absolute value is negative: $\frac{4 - 5x}{6} = -7$

Solve each equation:

(i) **Case 1:**

$\Rightarrow \quad \frac{4 - 5x}{6} = 7 \quad \Rightarrow \quad 4 - 5x = 42 \quad \Rightarrow \quad -5x = 38 \quad \Rightarrow \quad x = -\frac{38}{5}$

(ii) **Case 2:**

$\Rightarrow \quad \frac{4 - 5x}{6} = -7 \quad \Rightarrow \quad 4 - 5x = -42 \quad \Rightarrow \quad -5x = -46$

$\Rightarrow \quad x = \frac{46}{5} \quad \Rightarrow \quad x = -\frac{38}{5}, \frac{46}{5}$

$\therefore$ **Solution set** $= \left\{\left(-\frac{38}{5}, \frac{46}{5}\right)\right\}$

Q7. Explain, why the equation $|3x - 6| + 7 = 4$ has no solution.

Solution: $|3x - 6| + 7 = 4$

$\Rightarrow \quad |3x - 6| = 4 - 7 \quad \Rightarrow \quad |3x - 6| = -3$

Since the absolute value of non-zero integer is always positive.

Therefore absolute value cannot be negative, there is **no solution.**

Q8. Before the start of a professional basketball game, a basketball must be inflated to an air pressure of 8 pounds per square inch (psi) (Absolute error is the absolute deviation). Find the minimum and maximum air pressure acceptable for the basketball.

Solution:

- **Understand Absolute Deviation:**

  Absolute deviation (or absolute error) represents the amount of acceptable variation above and below the target value.

$\Rightarrow$ **Minimum = Target Value − Absolute Deviation**

$\Rightarrow$ **Maximum = Target Value + Absolute Deviation**

Let's assume the **absolute deviation** is given as $\mathbf{0.5}$ **psi.**

Then: **Absolute Deviation** $= \pm 0.5$

$\Rightarrow$ **Minimum Pressure** $= 8\text{ psi} - 0.5\text{ psi} = 7.5\text{ psi}$

$\Rightarrow$ **Maximum Pressure** $= 8\text{ psi} + 0.5\text{ psi} = 8.5\text{ psi}$

➤ **Solve the inequality and graph the solution (9-14).**

Q9. $|9x - 1| \le 10$

Solution: $|9x - 1| \le 10$

This inequality is equivalent to:

$\Rightarrow$ $-10 \le 9x - 1 \le 10$

We can solve this compound inequality by adding $\mathbf{1}$ to all parts.

$\Rightarrow$ $-10 + 1 \le 9x - 1 + 1 \le 10 + 1$

Which simplifies to:

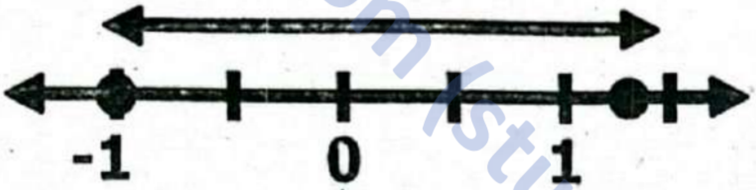
$\Rightarrow$ $-9 \le 9 \le 11$

Now, divide all parts by $\mathbf{9}$.

$\Rightarrow$ $\frac{-9}{9} \le \frac{9x}{9} \le \frac{11}{9}$ $\Rightarrow$ $-1 \le x \le \frac{11}{9}$

Thus, the solution set is $\left\{-1 \le x \le \frac{11}{9}\right\}$.

- **Graph of the solution:**



Q10. $|2x - 7| < 1$

Solution: $|2x - 7| < 1$

The inequality $|2x - 7| < 1$ is equivalent to the compound inequality.

$\Rightarrow$ $-1 < 2x - 7 < 1$

Adding $\mathbf{7}$ to all parts, we have:

$\Rightarrow$ $-1 + 7 < 2x - 7 + 7 < 1 + 7$

Which simplifies to:

$\Rightarrow$ $6 < 2x < 8$

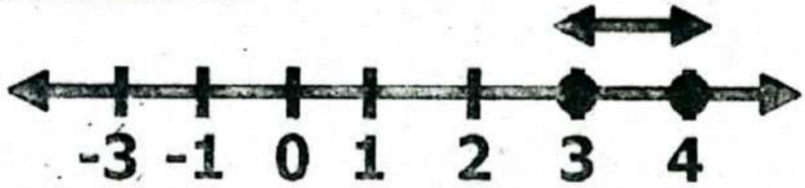
Dividing all parts by $\mathbf{2}$*, we get:*

$\Rightarrow$ $\frac{6}{2} < \frac{2x}{2} < \frac{8}{2}$ $\Rightarrow$ $3 < x < 4$

Thus, the solution set is $\{3 < x < 4\}$.

or the interval $(3, 4)$.

- **Graph of the solution:**



Q11. $5\left|\frac{1}{2}x+3\right|>5$

Solution: $5\left|\frac{1}{2}x+3\right|>5$

$\Rightarrow \quad \left|\frac{1}{2}x+3\right|>1$

Now, we consider two cases:

(i) **Case 1:**

$\Rightarrow \quad \frac{1}{2}x+3>1 \quad \Rightarrow \quad \frac{1}{2}x>1-3 \quad \Rightarrow \quad \frac{1}{2}x>-2 \quad \Rightarrow \quad x>-4$

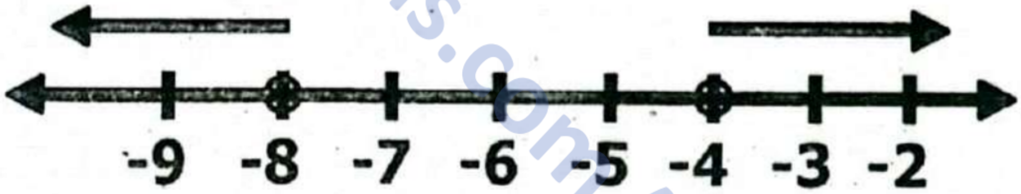
(ii) **Case 2:**

$\Rightarrow \quad \frac{1}{2}x+3<-1 \quad \Rightarrow \quad \frac{1}{2}x<-1-3 \quad \Rightarrow \quad \frac{1}{2}x<-4 \quad \Rightarrow \quad x<-8$

So the solution is $x<-8$ or $x>-4$.

Thus, the solution set is $\{x<-8$ or $x>-4\}$.

- **Graph of the solution:**



Q12. $3|14-x|>6$

Solution: $3|14-x|>6$

First, divide both sides by 3.

$\Rightarrow \quad |14-x|>2$

Now, we consider two cases:

(i) **Case 1:**

$\Rightarrow \quad 14-x>2 \quad \Rightarrow \quad -x>2-14 \quad \Rightarrow \quad -x>-12 \quad \Rightarrow \quad x<12$

(ii) **Case 2:**

$\Rightarrow \quad 14-x<-2 \quad \Rightarrow \quad -x<-2-14 \quad \Rightarrow \quad -x<-16 \quad \Rightarrow \quad x>16$

So the solution is $x<12$ or $x>16$.

Thus, the solution set is $\{x<12$ or $x>16\}$.

- **Graph of the solution:**

Q13. $\left|\frac{3x-2}{5}\right| \le 2$

Solution: $\left|\frac{3x-2}{5}\right| \le 2$

This inequality is equivalent to:

$\Rightarrow \quad -2 \le \frac{3x-2}{5} \le 2$

Multiply all parts of the inequality by 5.

$\Rightarrow \quad -10 \le 3x-2 \le 10 \quad \Rightarrow \quad -10+2 \le 3x \le 10+2$

$\Rightarrow \quad -8 \le 3x \le 12$

Divide all parts of the inequality by 3.

$\Rightarrow \quad -\frac{8}{3} \le x \le 4$

Thus, the solution set is $\left\{-\frac{8}{3} \le x \le 4\right\}$.

- **Graph of the solution:**

Q14. $\left|\frac{2-5x}{4}\right| \ge \frac{2}{3}$

Solution: $\left|\frac{2-5x}{4}\right| \ge \frac{2}{3}$

To solve this inequality, we consider two cases:

(i) **Case 1:**

$\Rightarrow \quad \frac{2-5x}{4} \ge \frac{2}{3}$

Multiply both sides by 4.

$\Rightarrow \quad 2-5x \ge \frac{8}{3}$

Subtract 2 from both sides.

$\Rightarrow \quad -5x \ge \frac{8}{3} - 2 = \frac{8}{3} - \frac{6}{3} = \frac{2}{3}$

Divide both sides by -5 (and reverse the inequality sign):

$\Rightarrow \quad x \le \frac{2}{3} \cdot \frac{-1}{5} = -\frac{2}{15}$

(ii) **Case 2:**

$\Rightarrow \quad \frac{2-5x}{4} \le -\frac{2}{3}$

Multiply both sides by 4. $\Rightarrow$ $2-5x \le -\frac{8}{3}$

Subtract 2 from both sides. $\Rightarrow$ $-5x \le -\frac{8}{3} - 2 = -\frac{8}{3} - \frac{6}{3} = -\frac{14}{3}$

Divide both sides by -5 (and reverse the inequality sign):

$\Rightarrow$ $x \ge \frac{-14}{3} \times \frac{-1}{5} = \frac{14}{15}$

Thus, the solution to the inequality is $x \le -\frac{2}{15}$ or $x \ge \frac{14}{15}$.

Thus, the solution set is $\left\{x \le -\frac{2}{15} \text{ or } x \ge \frac{14}{15}\right\}$.

- **Graph of the solution:**

Q15. An essay contest requires that essay entries consists of 500 words with an absolute deviation of at most 30 words. What are the possible number of words that the essay can have?

Solution:

Let x be the number of words in the essay. The problem states that the essay should have 500 words with an absolute deviation of at most 30 words. This can be written as:

$\Rightarrow$ $|x-500| \le 30$

To solve this inequality, we consider two cases:

(i) Case 1:

$\Rightarrow$ $x - 500 \le 30$

Add 500 to both sides. $\Rightarrow$ $x \le 500 + 30 = 530$

(ii) Case 2:

$\Rightarrow$ $-(x-500) \le 30$

Multiply both sides by -1. $\Rightarrow$ $x - 500 \ge -30$

Add 500 to both sides. $\Rightarrow$ $x \ge 500 - 30 = 470$

Thus, the possible number of words that the essay can have is between 470 *and* 530, *inclusive*.

Thus, the solution set is $\{470 \le W \le 530\}$.

- **Graph of the solution:**