Exercise 4.1

➢ **Solve the inequality and graph the solution.**

Q1. $5x - 12 \leq 3x - 4$

Solution: $5x - 12 \leq 3x - 4$

Subtract $3x$ from both sides of the inequality to get the x terms on one side:

$\Rightarrow \quad 5x - 12 - 3x \leq 3x - 4 - 3x \quad \Rightarrow \quad 2x - 12 \leq -4$

Add 12 to both sides to isolate the x term:

$\Rightarrow \quad 2x - 12 + 12 \leq -4 + 12 \quad \Rightarrow \quad 2x \leq 8$

Divide both sides by 2 to solve for x.

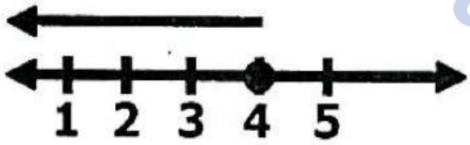
$\Rightarrow \quad \frac{2x}{2} \leq \frac{8}{2} \quad \Rightarrow \quad x \leq 4$

So the solution is $x \leq 4$.

This means x can be any value less than or equal to 4.

- **Graph of the solution:**

Now, let's represent this solution on a number line. We will use a closed circle at 4 to indicate that 4 is included in the solution, and shade to the left to indicate all values less than 4.



Q2. $1 - 8x \leq -4(2x - 1)$

Solution:

Let's solve the inequality $1 - 8x \leq -4(2x - 1)$ step by step:

Distribute the -4 on the right side of the inequality:

$\Rightarrow \quad 1 - 8x \leq -8x + 4$

Add $8x$ to both sides of the inequality:

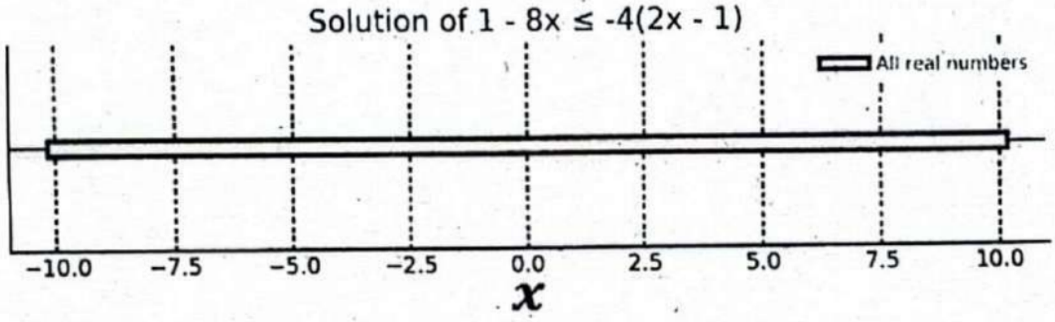
$\Rightarrow \quad 1 - 8x + 8x \leq -8x + 4 + 8x$

$\Rightarrow \quad 1 \leq 4$

Since $1 \leq 4$ is always true, this means that the inequality is true for all real numbers.

- **Graph of the solution:**

To represent this on a number line, you would shade the entire number line.



Q3. $\frac{-2}{3}x - 2 < \frac{1}{3}x + 8$

Solution: $\frac{-2}{3}x - 2 < \frac{1}{3}x + 8$

Add 2 to both sides of the inequality:

$\Rightarrow \quad \frac{-2}{3}x - 2 + 2 < \frac{1}{3}x + 8 + 2 \quad \Rightarrow \quad \frac{-2}{3}x < \frac{1}{3}x + 10$

Subtract $\frac{1}{3}x$ from both sides of the inequality:

$\Rightarrow \quad \frac{-2}{3}x - \frac{1}{3}x < \frac{1}{3}x + 10 - \frac{1}{3}x$

$\Rightarrow \quad \frac{-3}{3}x < 10 \quad \Rightarrow \quad -x < 10$

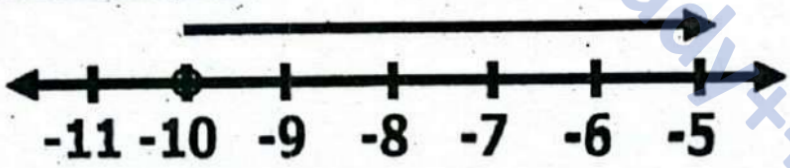
Multiply both sides by -1.

Remember to flip the inequality sign when multiplying or dividing by a negative number:

$\Rightarrow \quad (-1)(-x) > (-1)(10) \quad \Rightarrow \quad x > -10$

So the solution is $x > -10$.

- **Graph of the solution:**



Q4. $8 - \frac{4}{5}x > -14 + 2x$

Solution: $8 - \frac{4}{5}x > -14 + 2x$

Add $\frac{4}{5}x$ to both sides of the inequality:

$$\Rightarrow \quad 8-\frac{4}{5}x+\frac{4}{5}x>-14+2x+\frac{4}{5}x$$

$$\Rightarrow \quad 8>-14+\frac{10}{5}x+\frac{4}{5}x \qquad \Rightarrow \quad 8>-14+\frac{14}{5}x$$

Add **14** to both sides of the inequality:

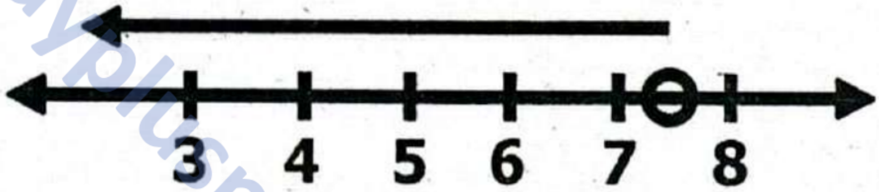
$$\Rightarrow \quad 8+14>-14+\frac{14}{5}x+14 \quad \Rightarrow \quad 22>\frac{14}{5}x$$

Multiply both sides by $\frac{5}{14}$ to isolate x.

$$\Rightarrow \quad \frac{5}{14}\cdot 22>\frac{5}{14}\cdot\frac{14}{5}x \quad \Rightarrow \quad \frac{110}{14}>x \quad \Rightarrow \quad \frac{55}{7}>x$$

So the solution is $x<\frac{55}{7}$.

- **Graph of the solution:**



Q5. $-0.6\,(x-5)\le 15$

Solution: $-0.6\,(x-5)\le 15$

Distribute the -0.6 on the left side of the inequality:

$$\Rightarrow \quad -0.6x+3\le 15$$

Subtract **3** from both sides of the inequality.

$$\Rightarrow \quad -0.6x+3-3\le 15-3 \quad \Rightarrow \quad -0.6x\le 12$$

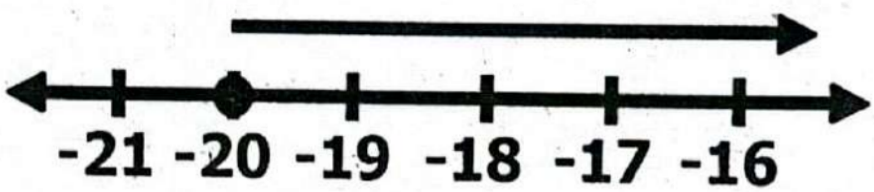
Divide both sides by -0.6.

Remember to flip the inequality sign when dividing by a negative number:

$$\Rightarrow \quad \frac{-0.6x}{-0.6}\ge\frac{12}{-0.6} \quad \Rightarrow \quad x\ge\frac{120}{-6} \quad \Rightarrow \quad x\ge -20$$

So the solution is $x\ge -20$.

- **Graph of the solution:**



Q6. $\frac{-1}{4}(x-12)>-2$

Solution: $\frac{-1}{4}(x-12)>-2$

Multiply both sides by -4.

When multiplying or dividing an inequality by a negative number, you must reverse the inequality sign.

$\Rightarrow \quad \frac{-1}{4}(x-12) > -2 \quad \Rightarrow \quad (-4)\cdot\frac{-1}{4}(x-12) < (-4)\cdot(-2)$

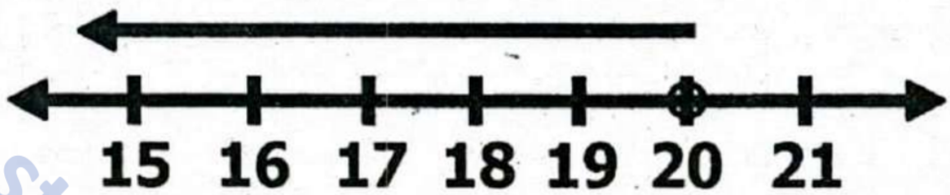
$\Rightarrow \quad x - 12 < 8$

Add **12** to both sides of the inequality.

$\Rightarrow \quad x - 12 + 12 < 8 + 12 \quad \Rightarrow \quad x < 20$

So, the solution to the inequality is $x < 20$.

- **Graph of the solution:**



➢ **Translate the phrase into an inequality. Then solve the inequality and graph the solution.**

Q7. Four more than the product of 3 and x is less than 40.

Solution:

The phrase "the product of **3** and x" translates to $3x$.

"Four more than the product of **3** and x" translates to $3x + 4$.

"is less than **40**" translates to < 40.

Putting it all together, the inequality is:

$\Rightarrow \quad 3x + 4 < 40$

Now, let's solve the inequality:

Subtract **4** from both sides.

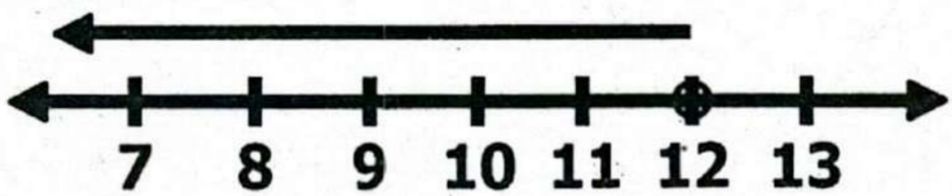
$\Rightarrow \quad 3x + 4 - 4 < 40 - 4 \Rightarrow \quad 3x < 36$

Divide both sides by **3**.

$\Rightarrow \quad \frac{3x}{3} < \frac{36}{3} \quad \Rightarrow \quad x < 12$

So the solution to the inequality is $x < 12$.

- **Graph of the solution:**



Q8. Twice the sum of x and 8 appears less than or equal to -2.

Solution:

"The sum of x and **8**" translates to $x + 8$.

"Twice the sum of x and **8**" translates to $2(x+8)$.

"Appears less than or equal to -2" translates to ≤ -2.

Putting it all together, the inequality is:

⇒ $2(x+8) \leq -2$

Now, let's solve the inequality:

Distribute the 2.

⇒ $2x+16 \leq -2$

Subtract 16 from both sides.

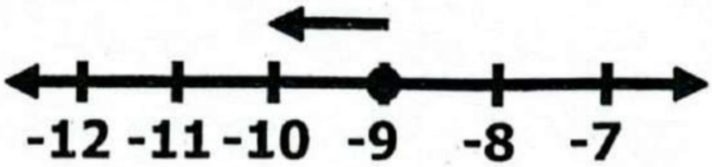
⇒ $2x+16-16 \leq -2-16 \quad \Rightarrow \quad 2x \leq -18$

Divide both sides by 2.

⇒ $\frac{2x}{2} \leq \frac{-18}{2} \quad \Rightarrow \quad x \leq -9$

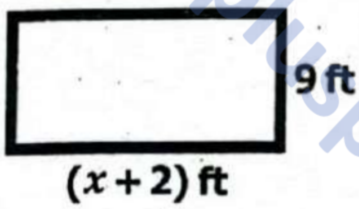
So the solution to the inequality is $x \leq -9$.

- **Graph of the solution:**



➤ **Write and solve an inequality to find the possible values of:**

Q9. **Area > 81 square feet**



Solution:

The area of a rectangle is given by the formula:

$$\text{Area} = \text{length} \times \text{width}$$

In this case, the length is $(x+2)$ **feet** and the width is **9 feet.** We are given that the area is greater than **81 square feet.**

So, we can write the inequality:

⇒ $9(x+2) > 81$

Now, let's solve the inequality:

Distribute the 9.

⇒ $9x+18 > 81 \quad \Rightarrow \quad 9x+18-18 > 81-18 \quad \Rightarrow \quad 9x > 63$

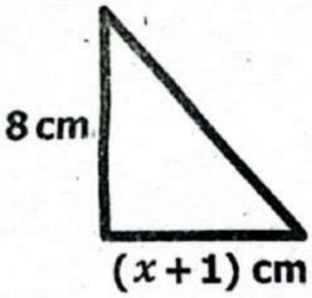
Divide both sides by 9.

⇒ $\frac{9x}{9} > \frac{63}{9} \quad \Rightarrow \quad x > 7$

So, the possible values of x are greater than 7.

The final answer is $x > 7$.

Q10. Area ≤ 44 square centimeters



Solution:

The area of a triangle is given by the formula:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

In this case, the base is $(x+1)$ cm and the height is 8 cm. We are given that the area is less than or equal to **44 square centimeters.** So, we can write the inequality:

$\Rightarrow \quad \frac{1}{2} \times 8 \times (x+1) \leq 44$

Simplify the equation.

$\Rightarrow \quad 4(x+1) \leq 44$

Now, let's solve the inequality:

Distribute the **4.**

$\Rightarrow \quad 4x + 4 \leq 44$

Subtract 4 from both sides.

$\Rightarrow \quad 4x + 4 - 4 \leq 44 - 4 \quad \Rightarrow \quad 4x \leq 40$

Divide both sides by **4.**

$\Rightarrow \quad \frac{4x}{4} \leq \frac{40}{4} \quad \Rightarrow \quad x \leq 10$

So, the possible values of x are less than or equal to **10.**

The final answer is $x \leq 10$.

➢ **<u>Solve each inequality:</u>**

Q11. $(x+1)(x-3) > 0$

Solution: $(x+1)(x-3) > 0$

To solve the inequality $(x+1)(x-3) > 0$, we need to determine the intervals where the expression $(x+1)(x-3)$ is positive.

First, find the critical points by setting each factor equal to zero:

$\Rightarrow \quad x+1=0 \quad \Rightarrow \quad x=-1$

$\Rightarrow \quad x-3=0 \quad \Rightarrow \quad x=3$

These critical points divide the number line into three intervals:

$\Rightarrow \quad x<-1 \quad \Rightarrow \quad -1<x<3 \quad \Rightarrow \quad x>3$

Now, we test a value from each interval to see if the inequality holds:

- $x < -1$:
  Let $x = -2$. Then $(-2+1)(-2-3) = (-1)(-5) = 5 > 0$.
  So the inequality holds for $x < -1$.
- $-1 < x < 3$:
  Let $x = 0$. Then $(0+1)(0-3) = (1)(-3) = -3 < 0$.
  So the inequality does not hold for $-1 < x < 3$.
- $x > 3$:
  Let $x = 4$. Then $(4+1)(4-3) = (5)(1) = 5 > 0$.
  So the inequality holds for $x > 3$.

Therefore, the solution to the inequality $(x+1)(x-3) > 0$ is $x < -1$ or $x > 3$.

The final answer is $x < -1$ or $x > 3$.

Q12. $(x-9)(x+1) < 0$

Solution: $(x-9)(x+1) < 0$

To solve the inequality $(x-9)(x+1) < 0$, we need to find the intervals where the expression $(x-9)(x+1)$ is negative.

First, find the critical points by setting each factor equal to zero:

$\Rightarrow \quad x-9=0 \quad \Rightarrow \quad x=9$

$\Rightarrow \quad x+1=0 \quad \Rightarrow \quad x=-1$

These critical points divide the number line into three intervals:

$\Rightarrow \quad x < -1 \quad \Rightarrow \quad -1 < x < 9 \quad \Rightarrow \quad x > 9$

Now, we test a value from each interval to see if the inequality holds:

- $x < -1$:
  Let $x = -2$. Then $(-2-9)(-2+1) = (-11)(-1) = 11 > 0$.
  So the inequality does not hold for $x < -1$.
- $-1 < x < 9$:
  Let $x = 0$. Then $(0-9)(0+1) = (-9)(1) = -9 < 0$.
  So the inequality holds for $-1 < x < 9$.
- $x > 9$:
  Let $x = 10$. Then $(10-9)(10+1) = (1)(11) = 11 > 0$.
  So the inequality does not hold for $x > 9$.

Therefore, the solution to the inequality $(x-9)(x+1) < 0$ is $-1 < x < 9$.

The final answer is $-1 < x < 9$.

Q13. $x^2 - x - 90 > 0$

Solution:

To solve the inequality $x^2 - x - 90 > 0$, we first need to find the roots of the corresponding quadratic equation $x^2 - x - 90 = 0$.

We can factor the quadratic expression:

$\Rightarrow \quad x^2 - x - 90 = (x-10)(x+9)$

So, the roots are $x = 10$ and $x = -9$.
These roots divide the number line into three intervals:
$\Rightarrow \quad x < -9 \quad \Rightarrow \quad -9 < x < 10 \quad \Rightarrow \quad x > 10$
Now, we test a value from each interval to see if the inequality holds:

- $x < -9$:
  Let $x = -10$. Then $(-10)^2 - (-10) - 90 = 100 + 10 - 90 = 20 > 0$.
  So the inequality holds for $x < -9$.
- $-9 < x < 10$:
  Let $x = 0$. Then $(0)^2 - (0) - 90 = -90 < 0$.
  So the inequality does not hold for $-9 < x < 10$.
- $x > 10$:
  Let $x = 11$. Then $(11)^2 - (11) - 90 = 121 - 11 - 90 = 20 > 0$.
  So the inequality holds for $x > 10$.

Therefore, the solution to the inequality $x^2 - x - 90 > 0$ is $x < -9$ or $x > 10$.
The final answer is $x < -9$ or $x > 10$.

Q14. $x^2 + 4x - 21 < 0$
Solution: $x^2 + 4x - 21 < 0$
To solve the inequality $x^2 + 4x - 21 < 0$, we first need to find the roots of the corresponding quadratic equation $x^2 + 4x - 21 = 0$.
We can factor the quadratic expression:
$\Rightarrow \quad x^2 + 4x - 21 = (x + 7)(x - 3)$
So, the roots are $x = -7$ and $x = 3$.
These roots divide the number line into three intervals:
$\Rightarrow \quad x < -7 \quad \Rightarrow \quad -7 < x < 3 \quad \Rightarrow \quad x > 3$
Now, we test a value from each interval to see if the inequality holds:

- $x < -7$:
  Let $x = -8$. Then $(-8)^2 + 4(-8) - 21 = 64 - 32 - 21 = 11 > 0$.
  So the inequality does not hold for $x < -7$.
- $-7 < x < 3$:
  Let $x = 0$. Then $(0)^2 + 4(0) - 21 = -21 < 0$.
  So the inequality holds for $-7 < x < 3$.
- $x > 3$:
  Let $x = 4$. Then $(4)^2 + 4(4) - 21 = 16 + 16 - 21 = 11 > 0$.
  So the inequality does not hold for $x > 3$.

Therefore, the solution to the inequality $x^2 + 4x - 21 < 0$ is $-7 < x < 3$.
The final answer is $-7 < x < 3$.

Q15. $x^2 + 8x + 16 \geq 0$
Solution: $x^2 + 8x + 16 \geq 0$
To solve the inequality $x^2 + 8x + 16 \geq 0$, we first need to analyze the corresponding quadratic expression $x^2 + 8x + 16$.

We can factor the quadratic expression:

$\Rightarrow \quad x^2 + 8x + 16 = (x+4)(x+4) = (x+4)^2$

So, the quadratic expression can be written as $(x+4)^2$. Since any real number squared is non-negative, $(x+4)^2$ is always greater than or equal to 0 for any real number x.

$\Rightarrow \quad (x+4)^2 \geq 0$

The inequality $(x+4)^2 \geq 0$ holds for all real numbers x.

Therefore, the solution to the inequality $x^2 + 8x + 16 \geq 0$ is all real numbers.

The final answer is <u>**all real numbers**</u> ; $x \neq 4$.

Q16. $9x^2 - 6x + 1 \leq 0$

Solution: $9x^2 - 6x + 1 \leq 0$

To solve the inequality $9x^2 - 6x + 1 \leq 0$, we first need to analyze the corresponding quadratic expression:

$\Rightarrow \quad 9x^2 - 6x + 1$

We can factor the quadratic expression:

$\Rightarrow \quad 9x^2 - 6x + 1 = (3x-1)(3x-1) = (3x-1)^2$

So, the quadratic expression can be written as $(3x-1)^2$. Since any real number squared is non-negative, $(3x-1)^2$ is always greater than or equal to 0 for any real number x.

$\Rightarrow \quad (3x-1)^2 \geq 0$

However, we are looking for when $(3x-1)^2 \leq 0$. The only way for a square to be less than or equal to 0 is for it to be equal to 0.

So we need to solve:

$\Rightarrow \quad (3x-1)^2 = 0 \quad \Rightarrow \quad 3x - 1 = 0 \quad \Rightarrow \quad 3x = 1 \quad \Rightarrow \quad x = \frac{1}{3}$

Therefore, the solution to the inequality $9x^2 - 6x + 1 \leq 0$ is $x = \frac{1}{3}$.

Q17. Mr. Khalid has a field and want to make a rectangular garden with perimeter of 68 ft. He would like the area of a garden to be at least 240 square feet. What would the width of the garden be?

Solution:

Let, l be the length and w be the width of the rectangular garden. The perimeter of the rectangular garden is given by $P = 2l + 2w$. We are given that the perimeter is 68 ft, so we have:

$\Rightarrow \quad 2l + 2w = 68$

Dividing by 2, we get:

$\Rightarrow \quad l + w = 34$

We can express the length in terms of the width:

$\Rightarrow \quad l = 34 - w$

The area of the rectangular garden is given by $A = lw$. We are given that the area must be at least 240 square feet, so we have:

$\Rightarrow$ $lw \geq 240$

Substitute $l = 34 - w$.

$\Rightarrow$ $(34 - w)w \geq 240$ $\Rightarrow$ $34w - w^2 \geq 240$

$\Rightarrow$ $0 \geq w^2 - 34w + 240$ $\Rightarrow$ $w^2 - 34w + 240 \leq 0$

Now, we need to find the roots of the quadratic equation:

$\Rightarrow$ $w^2 - 34w + 240 = 0$

We can factor the quadratic expression:

$\Rightarrow$ $(w - 10)(w - 24) = 0$

So, the roots are $w = 10$ and $w = 24$.

These roots divide the number line into three intervals:

$\Rightarrow$ $w < 10$ $\Rightarrow$ $10 < w < 24$ $\Rightarrow$ $w > 24$

We are looking for where $w^2 - 34w + 240 \leq 0$.

We test a value from each interval.

- $w < 10$:
  Let $w = 0$. Then $(0)^2 - 34(0) + 240 = 240 > 0$.
  So the inequality does not hold for $w < 10$.
- $10 < w < 24$:
  Let $w = 15$. Then $(15)^2 - 34(15) + 240 = 225 - 510 + 240 = -45 < 0$.
  So the inequality holds for $10 < w < 24$.
- $w > 24$:
  Let $w = 25$. Then $(25)^2 - 34(25) + 240 = 625 - 850 + 240 = 15 > 0$.
  So the inequality does not hold for $w > 24$.

Since we are looking for $w^2 - 34w + 240 \leq 0$, we include the roots in our interval. Therefore, the solution to the inequality is $10 \leq w \leq 24$.

The width of the garden can be between 10 ft and 24 ft, inclusive.

The final answer is $10 \leq w \leq 24$.

Q18. Are $-2, -1, 0, 1, 2$ solution of the inequality $2x^2 + 3x + 1 \leq 0$.

Solution:

To determine if the given values $-2, -1, 0, 1, 2$ are solutions to the inequality $2x^2 + 3x + 1 \leq 0$, we need to substitute each value of x into the inequality and check if it holds true.

- $x = -2$:

$\Rightarrow$ $2(-2)^2 + 3(-2) + 1 = 2(4) - 6 + 1 = 8 - 6 + 1 = 3$

Since $3 > 0$, -2 is not a solution.

- $x = -1$:

$\Rightarrow$ $2(-1)^2 + 3(-1) + 1 = 2(1) - 3 + 1 = 2 - 3 + 1 = 0$

Since $0 \leq 0$, -1 is a solution.

- $x = 0$:

$\Rightarrow$ $2(0)^2 + 3(0) + 1 = 0 + 0 + 1 = 1$

Since $1 > 0$, 0 is not a solution.

- $x = 1$:

$\Rightarrow$ $2(1)^2 + 3(1) + 1 = 2(1) + 3 + 1 = 2 + 3 + 1 = 6$

Since $6 > 0$, 1 is not a solution.

- $x = 2$:

$\Rightarrow$ $2(2)^2 + 3(2) + 1 = 2(4) + 6 + 1 = 8 + 6 + 1 = 15.$

Since $15 > 0$, 2 is not a solution.

Therefore, only $x = -1$ is a solution to the inequality $2x^2 + 3x + 1 \leq 0.$

There are no solutions except $-1.$

Q19. The stopping distance $d(x)$ (in meters) of a car traveling at x km/h is modeled by $d(x) = 0.05x^2 + 0.2x$. If the maximum stopping distance allowed in a school zone is 35 meters, find the speed range that ensures the stopping distance does not exceed this limit.

Solution: $d(x) = 0.05x^2 + 0.2x$

We are given the stopping distance function $d(x) = 0.05x^2 + 0.2x$, where x is the speed in **km/h.** We want to find the speed range such that the stopping distance does not exceed **35 meters.**

So, we need to solve the inequality:

$\Rightarrow$ $0.05x^2 + 0.2x \leq 35$

First, let's rewrite the inequality:

$\Rightarrow$ $0.05x^2 + 0.2x - 35 \leq 0$

To solve this inequality, we can first find the roots of the quadratic equation:

$\Rightarrow$ $0.05x^2 + 0.2x - 35 = 0$

We can multiply the equation by 20 to get rid of the decimals:

$\Rightarrow$ $x^2 + 4x - 700 = 0$

Now we can use the quadratic formula to find the roots:

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $a = 1$, $b = 4$, and $c = -700.$

$$\Rightarrow \quad x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-700)}}{2(1)} \quad \Rightarrow \quad x = \frac{-4 \pm \sqrt{16 + 2800}}{2}$$

$$\Rightarrow \quad x = \frac{-4 \pm \sqrt{2816}}{2} \quad \Rightarrow \quad x = \frac{-4 \pm 53.066}{2}$$

The two roots are:

$$\Rightarrow \quad x_1 = \frac{-4 - 53.066}{2} = \frac{-57.066}{2} \approx -28.533$$

$\Rightarrow \quad x_2 = \frac{-4 + 53.066}{2} = \frac{49.066}{2} \approx 24.54$

Since speed cannot be negative, we only consider the positive root:

$\Rightarrow \quad x_2 \approx 24.54$

The quadratic expression is less than or equal to **0** between the roots.

Therefore, the speed range is:

$\Rightarrow \quad -28.533 \leq x \leq 24.54$

Since speed must be non-negative, we have:

$\Rightarrow \quad 0 \leq x \leq 24.54$

Therefore, the speed range that ensures the stopping distance does not exceed **35 meters** is approximately **0 km/h** to **24.54 km/h.**

The final answer is $0 \leq x \leq 24.54$.

Q20. A rectangular garden is to be enclosed with a fence. The total length of the fence is at most 60 meters and the width x of the garden is twice its length. Find the possible dimensions of the garden.

Solution:

Let, l be the length and w be the width of the rectangular garden. We are given that the width w is twice its length, so $w = 2l$.

The perimeter of the rectangle is $P = 2l + 2w$. The total length of the fence is at most **60 meters,** so $P \leq 60$.

We have $2l + 2w \leq 60$. Since $w = 2l$, we can substitute this into the inequality:

$\Rightarrow \quad 2l + 2(2l) \leq 60$

$\Rightarrow \quad 2l + 4l \leq 60 \quad \Rightarrow \quad 6l \leq 60 \quad \Rightarrow \quad l \leq \frac{60}{6} \quad \Rightarrow \quad l \leq 10$

So the length is at most **10 meters.**

Since the width is twice the length, we have:

$\Rightarrow \quad w = 2l \leq 2(10) \quad \Rightarrow \quad w \leq 20$

So the width is at most **20 meters.**

Since the length and width must be positive, we have $l > 0$ and $w > 0$.

Therefore, the possible dimensions are:

$\Rightarrow \quad 0 < l \leq 10 \quad \Rightarrow \quad 0 < w \leq 20$

Since $w = 2l$, the possible dimensions are:

$\Rightarrow \quad 0 < l \leq 10 \text{ meters} \quad \Rightarrow \quad w = 2l$

So, $0 < w \leq 20$ **meters**

The final answer is:

$\Rightarrow \quad 0 < \text{length} \leq 10 \text{ meters}$ and $0 < \text{width} \leq 20 \text{ meters}$

$\Rightarrow \quad L : x \leq 10 \text{ m} \;,\quad W : 2x \leq 20 \text{ m}$

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