Exercise 2.4

Q1. Find the sum and product of the roots of the quadratic equation.

(i) $x^2 - 5x + 2 = 0$

Solution: $x^2 - 5x + 2 = 0$

$a = 1, \quad b = -5, \quad c = 2$

Sum of the roots $= \alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{1} = 5$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{2}{1} = 2$

So, the sum and product of the roots are $S = 5$ and $P = 2$, respectively.

(ii) $-4x^2 - 6x - 2 = 0$

Solution: $-4x^2 - 6x - 2 = 0$

$a = -4, \quad b = -6, \quad c = -2$

Sum of the roots $= \alpha + \beta = \frac{-b}{a} = \frac{-(-6)}{-4} = \frac{-3}{2}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{-2}{-4} = \frac{1}{2}$

So, the sum and product of the roots are $S = \frac{-3}{2}$ and $P = \frac{1}{2}$, respectively.

(iii) $5x^2 - 2x + 2 = 0$

Solution: $5x^2 - 2x + 2 = 0$

$a = 5, \quad b = -2, \quad c = 2$

Sum of the roots $= \alpha + \beta = \frac{-b}{a} = \frac{-(-2)}{5} = \frac{2}{5}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{2}{5}$

So, the sum and product of the roots are $S = \frac{2}{5}$ and $P = \frac{2}{5}$, respectively.

(iv) $-4x^2 - 8x - 9 = 0$

Solution: $-4x^2 - 8x - 9 = 0$

$a = -4, \quad b = -8, \quad c = -9$

Sum of the roots $= \alpha + \beta = \frac{-b}{a} = \frac{-(-8)}{-4} = \frac{-8}{4} = -2$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{-9}{-4} = \frac{9}{4}$

So, the sum and product of the roots are $S = -2$ and $P = \frac{9}{4}$, respectively.

(v) $16y^2 - 17y - 12 = 0$

Solution: $16y^2 - 17y - 12 = 0$

$a = 16, \quad b = -17, \quad c = -12$

Sum of the roots $= \alpha + \beta = \frac{-b}{a} = \frac{-(-17)}{16} = \frac{17}{16}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{-12}{16} = \frac{-3}{4}$

So, the sum and product of the roots are $S = \frac{17}{16}$ and $P = \frac{-3}{4}$, respectively.

(vi) $0.3x^2 - 7.7x + 1.8 = 0$

Solution: $0.3x^2 - 7.7x + 1.8 = 0$

$a = 0.3, \quad b = -7.7, \quad c = 1.8$

Sum of the roots $= \alpha + \beta = \frac{-b}{a} = \frac{-(-7.7)}{0.3} = 25.67$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{1.8}{0.3} = \frac{18}{3} = 6$

So, the sum & product of the roots are $S = 25.67$ and $P = 6$, respectively.

Q2. <u>Form a quadratic equation with roots:</u>

(i) $1, -\frac{8}{3}$

Solution: Given roots are: $1, \ -\frac{8}{3}$

$\Rightarrow$ $\text{S} = \text{Sum of roots} = 1 + \left(-\frac{8}{3}\right) = 1 - \frac{8}{3} = \frac{3}{3} - \frac{8}{3} = -\frac{5}{3}$

$\Rightarrow$ $\text{P} = \text{Product of the roots} = (1)\left(-\frac{8}{3}\right) = -\frac{8}{3}$

The required equation is:

$\Rightarrow$ $x^2 - (\text{sum of roots})\, x + \text{product of roots} = 0$

$\Rightarrow$ $x^2 - Sx + P = 0 \qquad \dots\dots\dots(1)$

Substituting these values of S and P in equation (1), we have:

$\Rightarrow$ $x^2 - \frac{5}{3}x - \frac{8}{3} = 0$

To get rid of the fractions, we can multiply the entire equation by **3**.

Thus, the quadratic equation is $3x^2 - 5x - 8 = 0$.

(ii) $\sqrt{3}, \ 2\sqrt{3}$

Solution:

Given roots are: $\sqrt{3}, \ 2\sqrt{3}$

$\Rightarrow$ $\text{S} = \text{Sum of roots} = \sqrt{3} + 2\sqrt{3} = 3\sqrt{3}$

$\Rightarrow$ $\text{P} = \text{Product of the roots} = (\sqrt{3})(2\sqrt{3}) = 2 \times 3 = 6$

The required equation is:

$\Rightarrow \quad x^2 - (\text{sum of roots})\,x + \text{product of roots} = 0$

$\Rightarrow \quad x^2 - Sx + P = 0 \qquad \ldots\ldots\ldots(1)$

Substituting these values of S and P in equation (1), we have:

$\Rightarrow \quad x^2 - 3\sqrt{3}x + 6 = 0$

Thus, the quadratic equation is $x^2 - 3\sqrt{3}x + 6 = 0$.

(iii) $2 + \sqrt{3}\,,\ 2 - \sqrt{3}$

Solution:

Given roots are: $2 + \sqrt{3}\,,\ 2 - \sqrt{3}$

$\Rightarrow \quad S = \text{Sum of roots} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

$\Rightarrow \quad P = \text{Product of the roots} = (2 + \sqrt{3})(2 - \sqrt{3})$

$= (2)^2 - (\sqrt{3})^2 = 4 - 3 = 1$

The required equation is:

$\Rightarrow \quad x^2 - (\text{sum of roots})\,x + \text{product of roots} = 0$

$\Rightarrow \quad x^2 - Sx + P = 0 \qquad \ldots\ldots\ldots(1)$

Substituting these values of S and P in equation (1), we have:

$\Rightarrow \quad x^2 - 4x + 1 = 0$

Thus, the quadratic equation is $x^2 - 4x + 1 = 0$.

(iv) $5i\,,\ -5i$

Solution:

Given roots are: $5i\,,\ -5i$

$\Rightarrow \quad S = \text{Sum of roots} = 5i - 5i = 0$

$\Rightarrow \quad P = \text{Product of the roots} = (5i)(5i)$

$= -25i^2 = -25(-1) = 25 \quad ; \quad [\because\ i^2 = -1]$

The required equation is:

$\Rightarrow \quad x^2 - (\text{sum of roots})\,x + \text{product of roots} = 0$

$\Rightarrow \quad x^2 - Sx + P = 0 \qquad \ldots\ldots\ldots(1)$

Substituting these values of S and P in equation (1), we have:

$\Rightarrow \quad x^2 - 0x + 25 = 0$

Thus, the quadratic equation is $x^2 + 25 = 0$.

(v) $7 + 2i\,,\ 7 - 2i$

Solution:

Given roots are: $7 + 2i\,,\ 7 - 2i$

$\Rightarrow \quad S = \text{Sum of roots} = 7 + 2i + 7 - 2i = 14$

$\Rightarrow \quad P = \text{Product of the roots} = (7 + 2i)(7 - 2i)$

$= (7)^2 - (2i)^2 = 49 - 4i^2 = 49 - 4(-1)$

$= 49 + 4 = 53 \qquad ; \qquad [\because\ i^2 = -1]$

The required equation is:

$\Rightarrow \quad x^2 - (\text{sum of roots})\,x + \text{product of roots} = 0$

$\Rightarrow \quad x^2 - Sx + P = 0 \quad \ldots\ldots\ldots (1)$

Substituting these values of **S** and **P** in equation (1), we have:

$\Rightarrow \quad x^2 - 14x + 53 = 0$

Thus, the quadratic equation is $x^2 - 14x + 53 = 0.$

Q3. If S_1, S_2 are the roots of $3x^2 - 2x + 4 = 0$, find the value of:

(i) $\frac{1}{S_1{}^2} + \frac{1}{S_2{}^2}$

Solution:

Since $S_1,\ S_2$ are the roots of $3x^2 - 2x + 4 = 0.$

$a = 3, \quad b = -2, \quad c = 4$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-(-2)}{3} = \frac{2}{3}$

$\Rightarrow$ Product of roots $= S_1 S_2 = \frac{c}{a} = \frac{4}{3}$

Now,

$$\Rightarrow \quad \frac{1}{S_1{}^2} + \frac{1}{S_2{}^2} = \frac{S_2{}^2 + S_1{}^2}{S_1{}^2 S_2{}^2}$$

$$= \frac{S_1{}^2 + S_2{}^2 + 2S_1 S_2 - 2S_1 S_2}{(S_1 S_2)^2} = \frac{(S_1 + S_2)^2 - 2S_1 S_2}{(S_1 S_2)^2}$$

$$= \frac{\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)}{\left(\frac{4}{3}\right)^2} = \frac{\frac{4}{9} - \frac{8}{3}}{\frac{16}{9}} = \frac{\frac{4-24}{9}}{\frac{16}{9}} = \frac{4-24}{9} \times \frac{9}{16} = \frac{-20}{16} = \frac{-5}{4}$$

(ii) $S_1{}^2 + S_2{}^2$

Solution:

Since $S_1,\ S_2$ are the roots of $3x^2 - 2x + 4 = 0.$

$a = 3, \quad b = -2, \quad c = 4$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-(-2)}{3} = \frac{2}{3}$

$\Rightarrow$ Product of roots $= S_1 S_2 = \frac{c}{a} = \frac{4}{3}$

Now,

$$\Rightarrow \quad S_1{}^2 + S_2{}^2 = S_1{}^2 + S_2{}^2 + 2S_1 S_2 - 2S_1 S_2 = (S_1 + S_2)^2 - 2S_1 S_2$$

$$= \left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right) = \frac{4}{9} - \frac{8}{3} = \frac{4-24}{9} = \frac{-20}{9}$$

(iii) $2S_1 + 2S_2 + 4$

Solution:

Since $S_1,\ S_2$ are the roots of $3x^2 - 2x + 4 = 0.$

$a = 3, \quad b = -2, \quad c = 4$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-(-2)}{3} = \frac{2}{3}$

$\Rightarrow$ Product of roots $= S_1 S_2 = \frac{c}{a} = \frac{4}{3}$

Now, $2S_1 + 2S_2 + 4 = 2(S_1 + S_2 + 2)$

$$= 2\left(\frac{2}{3} + 2\right) = 2\left(\frac{2+6}{3}\right) = 2\left(\frac{8}{3}\right) = \frac{16}{3}$$

(iv) $\frac{1}{S_1} + \frac{1}{S_2}$

Solution:

Since S_1, S_2 are the roots of $3x^2 - 2x + 4 = 0$.

$a = 3, \quad b = -2, \quad c = 4$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-(-2)}{3} = \frac{2}{3}$

$\Rightarrow$ Product of roots $= S_1 S_2 = \frac{c}{a} = \frac{4}{3}$

Now, $\frac{1}{S_1} + \frac{1}{S_2} = \frac{S_1 + S_2}{S_1 S_2} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{2}{3} \times \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$

(v) $\frac{S_1}{S_2} + \frac{S_2}{S_1}$

Solution:

Since S_1, S_2 are the roots of $3x^2 - 2x + 4 = 0$.

$a = 3, \quad b = -2, \quad c = 4$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-(-2)}{3} = \frac{2}{3}$

$\Rightarrow$ Product of roots $= S_1 S_2 = \frac{c}{a} = \frac{4}{3}$

Now,

$$\Rightarrow \frac{S_1}{S_2} + \frac{S_2}{S_1} = \frac{{S_1}^2 + {S_2}^2}{S_1 S_2} = \frac{{S_1}^2 + {S_2}^2 + 2S_1 S_2 - 2S_1 S_2}{S_1 S_2} = \frac{(S_1 + S_2)^2 - 2S_1 S_2}{S_1 S_2}$$

$$= \frac{\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)}{\left(\frac{4}{3}\right)} = \frac{\frac{4}{9} - \frac{8}{3}}{\frac{4}{3}} = \frac{4 - 24}{9} \times \frac{3}{4} = \frac{-20}{9} \times \frac{3}{4} = \frac{-5}{3}$$

(vi) $S_1 {S_2}^2 + {S_1}^2 S_2$

Solution:

Since S_1, S_2 are the roots of $3x^2 - 2x + 4 = 0$.

$a = 3, \quad b = -2, \quad c = 4$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-(-2)}{3} = \frac{2}{3}$

$\Rightarrow$ Product of roots $= S_1 S_2 = \frac{c}{a} = \frac{4}{3}$

Now, $S_1 S_2{}^2 + S_1{}^2 S_2 = S_1 S_2(S_1 + S_2)$

$$= S_1 S_2(S_1 + S_2) = \frac{4}{3}\left(\frac{2}{3}\right) = \frac{8}{9}$$

(vii) $S_1{}^3 S_2 + S_1 S_2{}^3$

Solution:

Since S_1, S_2 are the roots of $3x^2 - 2x + 4 = 0$.

$a = 3$, $b = -2$, $c = 4$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-(-2)}{3} = \frac{2}{3}$

$\Rightarrow$ Product of roots $= S_1 S_2 = \frac{c}{a} = \frac{4}{3}$

Now, $S_1{}^3 S_2 + S_1 S_2{}^3 = S_1 S_2(S_1{}^2 + S_2)$

$$= S_1 S_2(S_1{}^2 + S_2{}^2 + 2S_1 S_2 - 2S_1 S_2)$$

$$= S_1 S_2[(S_1 + S_2)^2 - 2S_1 S_2]$$

$$= \frac{4}{3}\left[\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)\right] = \frac{4}{3}\left(\frac{4}{9} - \frac{8}{3}\right)$$

$$= \frac{4}{3}\left(\frac{4-24}{9}\right) = \frac{4}{3}\left(\frac{-20}{9}\right) = \frac{-80}{27}$$

(viii) $(S_1 - 3)(S_2 - 3)$

Solution:

Since S_1, S_2 are the roots of $3x^2 - 2x + 4 = 0$.

$a = 3$, $b = -2$, $c = 4$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-(-2)}{3} = \frac{2}{3}$

$\Rightarrow$ Product of roots $= S_1 S_2 = \frac{c}{a} = \frac{4}{3}$

Now,

$\Rightarrow$ $(S_1 - 3)(S_2 - 3) = S_1 S_2 - 3S_1 - 3S_2 + 9 = S_1 S_2 - 3(S_1 + S_2) + 9$

$$= \frac{4}{3} - 3\left(\frac{2}{3}\right) + 9 = \frac{4}{3} - \frac{6}{3} + \frac{9}{1} = \frac{4 - 6 + 27}{3} = \frac{25}{3}$$

Q4. If S_1, S_2 are the roots of $7x^2 + 10x + 7 = 0$, form the equations whose roots are:

(i) $S_1{}^2$, $S_2{}^2$

Solution:

Since S_1, S_2 are the roots of $7x^2 + 10x + 7 = 0$.

$a = 7$, $b = 10$, $c = 7$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-10}{7}$ ……… (1)

$\Rightarrow$ Product of roots $= S_1\,S_2 = \frac{c}{a} = \frac{7}{7} = 1$ .... (2)

Now, for the new roots $S_1{}^2$ and $S_2{}^2$.

$\Rightarrow$ Sum of new roots $= S = S_1{}^2 + S_2{}^2$

$$= S_1{}^2 + S_2{}^2 + 2S_1\,S_2 - 2S_1\,S_2$$
$$= (S_1 + S_2)^2 - 2S_1\,S_2 \quad ; \quad \text{[From (1) and (2)]}$$
$$= \left(\frac{-10}{7}\right)^2 - 2(1) = \frac{100}{49} - 2$$
$$= \frac{100-98}{49} = \frac{2}{49}$$

$\Rightarrow$ Product of new roots $= P = (S_1{}^2)(S_2{}^2) = (S_1\,S_2)^2$

$$= (1)^2 = 1$$

Now substituting the values of S and P in $x^2 - Sx + P = 0$, we get:

$$\Rightarrow \quad x^2 - \left(\frac{2}{49}\right)x + 1 = 0 \quad \Rightarrow \quad x^2 - \frac{2x}{49} + 1 = 0$$

Multiplying by 49 to clear the fraction, we get:

$$\Rightarrow \quad 49(x^2) - 49\left(\frac{2x}{49}\right) + 49(1) = 0 \quad \Rightarrow \quad 49x^2 - 2x + 49 = 0$$

(ii) $\frac{1}{S_1}, \frac{1}{S_2}$

Solution:

Since S_1, S_2 are the roots of $7x^2 + 10x + 7 = 0$.

$a = 7, \quad b = 10, \quad c = 7$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-10}{7}$ .... (1)

$\Rightarrow$ Product of roots $= S_1\,S_2 = \frac{c}{a} = \frac{7}{7} = 1$ .... (2)

Now, for the new roots $\frac{1}{S_1}$ and $\frac{1}{S_2}$.

$\Rightarrow$ Sum of new roots $= S = \frac{1}{S_1} + \frac{1}{S_2} = \frac{S_1 + S_2}{S_1\,S_2} = \frac{\frac{-10}{7}}{1} = \frac{-10}{7}$

$\Rightarrow$ Product of new roots $= P = \left(\frac{1}{S_1}\right)\left(\frac{1}{S_2}\right) = \left(\frac{1}{S_1\,S_2}\right) = \left(\frac{1}{1}\right) = 1$

Now substituting the values of S and P in $x^2 - Sx + P = 0$, we get:

$$\Rightarrow \quad x^2 - \left(\frac{-10}{7}\right)x + 1 = 0 \quad \Rightarrow \quad x^2 + \frac{10x}{7} + 1 = 0$$

Multiplying by 7 to clear the fraction, we get:

$$\Rightarrow \quad 7(x^2) + 7\left(\frac{10x}{7}\right) + 7(1) = 0 \quad \Rightarrow \quad 7x^2 + 10x + 7 = 0$$

$\Rightarrow$ Product of new roots $= P = \left(S_1 - \frac{1}{S_1}\right)\left(S_2 - \frac{1}{S_2}\right)$

$$= S_1 S_2 - (S_1 + S_2) + \frac{1}{S_1 S_2}$$

$$= 1 - \left(\frac{-10}{7}\right) + \frac{1}{1} = 1 + \frac{10}{7} + 1$$

$$= \frac{7}{7} + \frac{10}{7} + \frac{7}{7} = \frac{24}{7}$$

Now substituting the values of S and P in $x^2 - Sx + P = 0$, we get:

$\Rightarrow \quad x^2 - 0(x) + \frac{24}{7} = 0 \qquad \Rightarrow \qquad x^2 + \frac{24}{7} = 0$

Multiplying by 7 to clear the fraction, we get:

$\Rightarrow \quad 7(x^2) + 7\left(\frac{24}{7}\right) = 0 \qquad \Rightarrow \qquad 7x^2 + 24 = 0$

(v) $2S_1 + 1,\ 2S_2 + 1$

Solution:

Since S_1, S_2 are the roots of $7x^2 + 10x + 7 = 0$.

$a = 7, \quad b = 10, \quad c = 7$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-10}{7}$ ……… (1)

$\Rightarrow$ Product of roots $= S_1 S_2 = \frac{c}{a} = \frac{7}{7} = 1$ ……… (2)

Now, for the new roots $2S_1 + 1$ and $2S_2 + 1$.

$\Rightarrow$ Sum of new roots $= S = 2S_1 + 1 + 2S_2 + 1$

$$= 2S_1 + 2S_2 + 2 = 2(S_1 + S_2) + 2$$

$$= 2\left(\frac{-10}{7}\right) + 2 = \frac{-20 + 14}{7} = \frac{-6}{7}$$

$\Rightarrow$ Product of new roots $= P = (2S_1 + 1) + (2S_2 + 1)$

$$= 4S_1 S_2 + 2S_1 + 2S_2 + 1$$

$$= 4S_1 S_2 + 2(S_1 + S_2) + 1$$

$$= 4(1) + 2\left(\frac{-10}{7}\right) + 1$$

$$= 4 - \frac{20}{7} + 1 = \frac{28 - 20 + 7}{7} = \frac{15}{7}$$

Now substituting the values of S and P in $x^2 - Sx + P = 0$, we get:

$\Rightarrow \quad x^2 - \left(\frac{-6}{7}\right)x + \frac{15}{7} = 0 \qquad \Rightarrow \qquad x^2 + \frac{6x}{7} + \frac{15}{7} = 0$

Multiplying by 7 to clear the fraction, we get:

$\Rightarrow \quad 7(x^2) + 7\left(\frac{6x}{7}\right) + 7\left(\frac{15}{7}\right) = 0 \qquad \Rightarrow \qquad 7x^2 + 6x + 15 = 0$

(vi) $\frac{S_1}{S_2}, \frac{S_2}{S_1}$

Solution:

Since S_1, S_2 are the roots of $7x^2 + 10x + 7 = 0$.

$a = 7, \quad b = 10, \quad c = 7$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-10}{7}$ ……… (1)

$\Rightarrow$ Product of roots $= S_1 S_2 = \frac{c}{a} = \frac{7}{7} = 1$ ……… (2)

Now, for the new roots $\frac{S_1}{S_2}$ and $\frac{S_2}{S_1}$.

$\Rightarrow$ Sum of new roots $= S = \frac{S_1}{S_2} + \frac{S_2}{S_1} = \frac{S_1{}^2 + S_2{}^2}{S_1 S_2}$

$$= \frac{S_1{}^2 + S_2{}^2 + 2S_1 S_2 - 2S_1 S_2}{S_1 S_2} = \frac{(S_1 + S_2)^2 - 2S_1 S_2}{S_1 S_2}$$

$$= \frac{\left(\frac{-10}{7}\right)^2 - 2(1)}{1} = \frac{100}{49} - 2 = \frac{100 - 98}{49} = \frac{2}{49}$$

$\Rightarrow$ Product of new roots $= P = \left(\frac{S_1}{S_2}\right)\left(\frac{S_2}{S_1}\right) = \frac{S_1\beta}{S_1 S_2} = \frac{1}{1} = 1$

Now substituting the values of S and P in $x^2 - Sx + P = 0$, we get:

$\Rightarrow$ $x^2 - \left(\frac{2}{49}\right)x + 1 = 0 \qquad \Rightarrow x^2 - \frac{2x}{49} + 1 = 0$

Multiplying by 49 to clear the fraction, we get:

$\Rightarrow$ $49(x^2) - 49\left(\frac{2x}{49}\right) + 49(1) = 0 \quad \Rightarrow \quad 49x^2 - 2x + 49 = 0$

(vii) $S_1 + S_2, \frac{1}{S_1} + \frac{1}{S_2}$

Solution:

Since S_1, S_2 are the roots of $7x^2 + 10x + 7 = 0$.

$a = 7, \quad b = 10, \quad c = 7$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-10}{7}$ ……… (1)

$\Rightarrow$ *Product of roots* $= S_1 S_2 = \frac{c}{a} = \frac{7}{7} = 1$ ……… (2)

Now, for the new roots $S_1 + \beta$ and $\frac{1}{S_1} + \frac{1}{S_2}$.

$\Rightarrow$ Sum of new roots $= S = S_1 + S_2 + \frac{1}{S_1} + \frac{1}{S_2} = S_1 + S_2 + \left(\frac{S_2 + S_1}{S_1 S_2}\right)$

$$= \left(\frac{-10}{7}\right) + \left(\frac{\frac{-10}{7}}{1}\right) = -\frac{10}{7} - \frac{10}{7} = \frac{-10-10}{7} = \frac{-20}{7}$$

$\Rightarrow$ Product of new roots $= P = (S_1 + S_2)\left(\frac{1}{S_1} + \frac{1}{S_2}\right) = (S_1 + S_2)\left(\frac{S_2 + S_1}{S_1 S_2}\right)$

$$= \left(\frac{-10}{7}\right) + \left(\frac{\frac{-10}{7}}{1}\right) = \left(\frac{-10}{7}\right)\left(\frac{-10}{7}\right) = \frac{100}{49}$$

Now substituting the values of S and P in $x^2 - Sx + P = 0$, we get:

$\Rightarrow \quad x^2 - \left(\frac{-20}{7}\right)x + \frac{100}{49} = 0 \quad \Rightarrow \quad x^2 + \frac{20x}{7} + \frac{100}{49} = 0$

Multiplying by 49 to clear the fraction, we get:

$\Rightarrow \quad 49(x^2) + 49\left(\frac{20x}{7}\right) + 49\left(\frac{100}{49}\right) = 0 \quad \Rightarrow \quad 49x^2 + 140x + 100 = 0$

(viii) $S_1{}^2 + 1\,,\; S_2{}^2 + 1$

Solution:

Since $S_1,\ S_2$ are the roots of $7x^2 + 10x + 7 = 0$.

$a = 7\,, \quad b = 10\,, \quad c = 7$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-10}{7}$ ………(1)

$\Rightarrow$ Product of roots $= S_1 S_2 = \frac{c}{a} = \frac{7}{7}$ ………(2)

Now, for the new roots $S_1{}^2 + 1$ and $S_2{}^2 + 1$.

$\Rightarrow$ Sum of new roots $= S = S_1{}^2 + 1 + S_2{}^2 + 1 = S_1{}^2 + S_2{}^2 + 2$

$$= S_1{}^2 + S_2{}^2 + 2S_1 S_2 - 2S_1 S_2 + 2$$
$$= (S_1 + S_2)^2 - 2S_1 S_2 + 2$$
$$= \left(\frac{-10}{7}\right)^2 - 2(1) + 2 = \frac{100}{49}$$

$\Rightarrow$ Product of new roots $= P = (S_1{}^2 + 1)(S_2{}^2 + 1)$

$$= S_1{}^2 S_2{}^2 + S_1{}^2 + S_2{}^2 + 1$$
$$= (S_1 S_2)^2 + S_1{}^2 + S_2{}^2 + 2S_1 S_2 - 2S_1 S_2 + 1$$
$$= (S_1 S_2)^2 + (S_1 + S_2)^2 - 2S_1 S_2 + 1$$
$$= (1)^2 + \left(\frac{-10}{7}\right)^2 - 2(1) + 1 = \frac{100}{49}$$

Now substituting the values of S and P in $x^2 - Sx + P = 0$, we get:

$\Rightarrow \quad x^2 - \left(\frac{100}{49}\right)x + \frac{100}{49} = 0 \quad \Rightarrow \quad x^2 - \frac{100x}{49} + \frac{100}{49} = 0$

Multiplying by 49 to clear the fraction, we get:

$$\Rightarrow \quad 49(x^2) - 49\left(\frac{100x}{49}\right) + 49\left(\frac{100}{49}\right) = 0 \quad \Rightarrow \quad 49x^2 - 100x + 100 = 0$$

(ix) $S_1{}^2 + S_2{}^2,\ S_1 + S_2{}^2$

Solution:

Since S_1, S_2 are the roots of $7x^2 + 10x + 7 = 0.$

$a = 7, \quad b = 10, \quad c = 7$

$\Rightarrow$ Sum of the roots $= S_1 + S_2 = \frac{-b}{a} = \frac{-10}{7}$ ……… (1)

$\Rightarrow$ Product of roots $= S_1 S_2 = \frac{c}{a} = \frac{7}{7} = 1$ ……… (2)

Now, for the new roots $S_1{}^2 + S_2$ and $S_1 + S_2{}^2$.

$\Rightarrow$ Sum of new roots $= S = (S_1{}^2 + S_2) + (S_1 + S_2{}^2)$

$$= S_1 + S_2 + S_1{}^2 + S_2{}^2$$
$$= (S_1 + S_2) + (S_1{}^2 + S_2{}^2 + 2S_1 S_2 - 2S_1 S_2)$$
$$= (S_1 + S_2) + [(S_1 + S_2)^2 - 2S_1 S_2]$$
$$= \left(\frac{-10}{7}\right) + \left[\left(\frac{-10}{7}\right)^2 - 2(1)\right]$$
$$= \frac{-10}{7} + \left(\frac{100}{49} - 2\right) = \frac{-10}{7} + \left(\frac{100 - 98}{49}\right)$$
$$= \frac{-10}{7} + \frac{2}{49} = \frac{-70 + 2}{49} = \frac{-68}{49}$$

$\Rightarrow$ Product of new roots $= P = (S_1{}^2 + S_2)(S_1 + S_2{}^2)$

$$= S_1{}^3 + S_1{}^2 S_2{}^2 + S_1 S_2 + S_2{}^3$$
$$= (S_1 S_2)^2 + S_1 S_2 + S_1{}^3 + S_2{}^3$$
$$= (S_1 S_2)^2 + S_1 S_2 + [(S_1 + S_2)(S_1{}^2 + S_2{}^2 - S_1 S_2)]$$
$$= (S_1 S_2)^2 + S_1 S_2 + [(S_1 + S_2)(S_1{}^2 + S_2{}^2 + 2S_1 S_2 - 2S_1 S_2 - S_1 S_2)]$$
$$= (S_1 S_2)^2 + S_1 S_2 + [(S_1 + S_2)((S_1 + S_2)^2 - 3S_1 S_2)]$$
$$= (1)^2 + 1 + \left(\frac{-10}{7}\right)\left[\left(\frac{-10}{7}\right)^2 - 3(1)\right]$$
$$= 2 + \left[\frac{-10}{7}\left(\frac{100}{49} - 3\right)\right] = 2 + \left[\frac{-10}{7}\left(\frac{100 - 147}{49}\right)\right]$$
$$= 2 + \left[\frac{-10}{7}\left(\frac{-47}{49}\right)\right] = 2 + \left(\frac{470}{343}\right)$$
$$= \frac{686 + 470}{343} = \frac{1156}{343}$$