Exercise 2.3

Q1. Find the discriminant of the following quadratic equations.

(i) $x^2 + 6x - 27 = 0$

Solution: $x^2 + 6x - 27 = 0$

$a = 1, b = 6, c = -27$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= 6^2 - 4(1)(-27) = 36 + 108 = 144$

The discriminant of the quadratic equation $x^2 + 6x - 27 = 0$ is 144.

(ii) $x^2 - x - 12 = 0$

Solution: $x^2 - x - 12 = 0$

$a = 1, \quad b = -1, \quad c = -12$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (-1)^2 - 4(1)(-12) = 1 + 48 = 49$

The discriminant of the quadratic equation $x^2 - x - 12 = 0$ is 49.

(iii) $8x^2 + 2x + 1 = 0$

Solution: $8x^2 + 2x + 1 = 0$

$a = 8, \quad b = 2, \quad c = 1$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (2)^2 - 4(8)(1) = 4 - 32 = -28$

The discriminant of the quadratic equation $8x^2 + 2x + 1 = 0$ is -28.

(iv) $12x^2 - 11x - 15 = 0$

Solution: $12x^2 - 11x - 15 = 0$

$a = 12, \quad b = -11, \quad c = -15$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (-11)^2 - 4(12)(-15) = 121 + 720 = 841$

The discriminant of the quadratic equation $12x^2 - 11x - 15 = 0$ is 841.

Q2. Discuss the nature of roots of the following quadratic equations.

(i) $x^2 - 2x - 15 = 0$

Solution: $x^2 - 2x - 15 = 0$

$a = 1, \quad b = -2, \quad c = -15$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (-2)^2 - 4(1)(-15) = 4 + 60 = 64$

The value of discriminant is positive ($\text{Disc} = 64 > 0$) and a perfect square. So, the given equation has two real roots and they are rational and unequal.

(ii) $x^2 + 3x - 4 = 0$

Solution: $x^2 + 3x - 4 = 0$

$a = 1, \quad b = 3, \quad c = -4$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (3)^2 - 4(1)(-4) = 9 + 16 = 25$

The value of discriminant is positive ($\text{Disc} = 25 > 0$) and a perfect square. So, the given equation has two real roots and they are rational and unequal.

(iii) $12x^2 + x - 20 = 0$

Solution: $12x^2 + x - 20 = 0$

$a = 12, \quad b = 1, c = -20$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (1)^2 - 4(12)(-20) = 1 + 960 = 961$

The value of discriminant is positive ($\text{Disc} = 961 > 0$) and a perfect square. So, the given equation has two real roots and they are rational and unequal.

(iv) $x^2 + 2x + 8 = 0$

Solution: $x^2 + 2x + 8 = 0$

$a = 1, \quad b = 2, \quad c = 8$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (2)^2 - 4(1)(8) = 4 - 32 = -28$

The value of discriminant is negative ($\text{Disc} = -28 < 0$). So, the given equation has two imaginary roots.

(v) $x^2 + 3x - 9 = 0$

Solution: $x^2 + 3x - 9 = 0$

$a = 1, \quad b = 3, \quad c = -9$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (3)^2 - 4(1)(-9) = 9 + 36 = 45$

The value of discriminant is positive ($\text{Disc} = 45 > 0$) and a perfect square. So, the given equation has two real roots and they are irrational and unequal.

Q3. For what value of k, $9x^2 - kx + 16 = 0$ is a perfect square?

Solution: $9k^2 - kx + 16 = 0$

$a = 9, \quad b = -k, \quad c = 16$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (-k)^2 - 4(9)(16)$

$\Rightarrow$ $\text{Disc} = k^2 - 576$

Given expression is a perfect square. Therefore, the roots are rational and equal. Therefore, the discriminant is zero.

$\Rightarrow$ $\text{Disc} = k^2 - 576$

$\Rightarrow$ $0 = k^2 - 576 \quad \Rightarrow \quad k^2 = 576 \quad \Rightarrow \quad k = \pm 24$

Q4. If roots of $x^2 + kx + 9 = 0$ are equal, find k?

Solution: $x^2 + kx + 9 = 0$

$a = 1, \quad b = k, \quad c = 9$

$\Rightarrow \quad \text{Disc} = b^2 - 4ac$

$= (k)^2 - 4(1)(9)$

$\Rightarrow \quad \text{Disc} = k^2 - 36$

It is given that equation has equal roots (same root) so the discriminant is zero.

$\Rightarrow \quad \text{Disc} = k^2 - 36$

$\Rightarrow \quad 0 = k^2 - 36 \Rightarrow \quad k^2 = 36 \quad \Rightarrow \quad k = \pm 6$

Q5. Show that the roots of $2x^2 + (mx - 1)^2 = 3$, are equal if $3m^2 + 4 = 0$.

Solution:

$\Rightarrow \quad 2x^2 + (mx - 1)^2 = 3$

$\Rightarrow \quad 2x^2 + m^2x^2 - 2mx + 1 = 3$

$\Rightarrow \quad (2 + m^2)x^2 - 2mx + (1 - 3) = 0$

$\Rightarrow \quad (m^2 + 2)x^2 - 2mx - 2 = 0$

$a = m^2 + 2, \quad b = -2m, \quad c = -2$

$\Rightarrow \quad \text{Disc} = b^2 - 4ac$

$= (-2m)^2 - 4(m^2 + 2)(-2) = 4m^2 + 8m^2 + 16$

$\Rightarrow \quad \text{Disc} = 12m^2 + 16$

For the roots to be equal, the discriminant of this quadratic equation must be zero. The condition for the roots to be equal is **Disc = 0.**

$\Rightarrow \quad \text{Disc} = 12m^2 + 16$

$\Rightarrow \quad 0 = 12m^2 + 16 \quad \Rightarrow \quad 0 = 12m^2 + 16 \quad \Rightarrow \quad 3m^2 + 4 = 0$

Thus, if $3m^2 + 4 = 0$, the discriminant $\mathbf{D}$ is zero, which implies that the roots of the equation $2x^2 + (mx - 1)^2 = 3$ are equal.

Q6. <u>Find the value of "m" when roots of the following quadratic equations are equal.</u>

(i) $x^2 - 6x + m = 0$

Solution: $x^2 - 6x + m = 0$

$a = 1, \quad b = -6, \quad c = m$

$\Rightarrow \quad \text{Disc} = b^2 - 4ac = (-6)^2 - 4(1)(m)$

$\Rightarrow \quad \text{Disc} = 36 - 4m$

Since the roots are equal. Therefore, the discriminant is zero.

$\Rightarrow \quad \text{Disc} = 36 - 4m = 0$

$\Rightarrow \quad 36 - 4m = 0 \quad \Rightarrow \quad 36 = 4m \quad \Rightarrow \quad m = \frac{36}{4} \quad \Rightarrow \quad m = 9$

So, the value of m for which the roots of the equation are equal is $m = 9$.

(ii) $m^2x^2 + (2m+1)x + 1 = 0$

Solution: $m^2x^2 + (2m+1)x + 1 = 0$

$a = m^2, \quad b = 2m+1, \quad c = 1$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (2m+1)^2 - 4(m^2)(1) \quad ; \quad [\because (a+b)^2 = a^2 + b^2 + 2ab]$

$= (2m)^2 + (1)^2 + 2(2m)(1) - 4m^2$

$= 4m^2 + 1 + 4m - 4m^2$

$\Rightarrow$ $\text{Disc} = 1 + 4m$

Since the roots are equal. Therefore, the discriminant is zero.

$\Rightarrow$ $\text{Disc} = 1 + 4m = 0$

$\Rightarrow$ $1 + 4m = 0 \Rightarrow 4m = -1 \Rightarrow m = \frac{-1}{4}$

So, the value of m for which the roots of the equation are equal is $= \frac{-1}{4}$.

(iii) $(m+3)x^2 + (m+1)x + m + 1 = 0$

Solution: $(m+3)x^2 + (m+1)x + m + 1 = 0$

$a = m+3, \quad b = m+1, \quad c = m+1$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (m+1)^2 - 4(m+3)(m+1) \ ; [\because (a+b)^2 = a^2 + b^2 + 2ab]$

$= (m)^2 + (1)^2 + 2(m)(1) - 4(m^2 + m + 3m + 3)$

$= m^2 + 1 + 2m - 4(m^2 + 4m + 3)$

$= m^2 + 1 + 2m - 4m^2 - 16m - 12$

$= -3m^2 - 14m - 11$

$= -1(3m^2 + 14m + 11)$

$\Rightarrow$ $\text{Disc} = 3m^2 + 14m + 11$

Since the roots are equal. Therefore, the discriminant is zero.

$\Rightarrow$ $\text{Disc} = 3m^2 + 14m + 11 = 0$

$\Rightarrow$ $3m^2 + 14m + 11 = 0$

$\Rightarrow$ $3m^2 + 11m + 3m + 11 = 0$

$\Rightarrow$ $m(3m+11) + 1(3m+11) = 0$

$\Rightarrow$ $(m+1)(3m+11) = 0$

$\Rightarrow$ $m + 1 = 0 \quad ; \quad 3m + 11 = 0$

$\Rightarrow$ $m = -1 \quad ; \quad 3m = -11$

$\Rightarrow$ $m = -1 \quad ; \quad m = \frac{-11}{3}$

Q7. Show that the roots of the equation $(a^2+b^2)x^2 + 2(ac+bd)x + c^2 + d^2 = 0$ are imaginary. Moreover, it shows repeated roots if $ad = bc$.

Solution:

$(a^2 + b^2)\,x^2 + 2(ac + bd)\,x + c^2 + d^2 = 0$

$a = a^2 + b^2, \quad b = 2(ac + bd), \quad c = c^2 + d^2$

$\Rightarrow$ Disc $= b^2 - 4ac$

$= [2(ac+bd)]^2 - 4(a^2+b^2)(c^2+d^2)$

$= 4(a^2c^2 + b^2d^2 + 2abcd) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$

$= 4a^2c^2 + 4b^2d^2 + 8abcd - 4a^2c^2 - 4a^2d^2 - 4b^2c^2 - 4b^2d^2$

$= -4a^2d^2 - 4b^2c^2 + 8abcd = -(4a^2d^2 + 4b^2c^2 - 8abcd)$

$= -[(2ad)^2 + (2bc)^2 - 2(2ad)(2bc)]$

$[\because \quad (a-b)^2 = a^2 + b^2 - 2ab]$

$= -(2ad - 2bc)^2 < 0$

$\Rightarrow$ $b^2 - 4ac < 0$

So, roots are imaginary.

For repeated roots, put $ad = bc$.

$\Rightarrow$ $b^2 - 4ac = -(2ad - 2bc)^2 = -(2bc - 2bc)^2$

$\Rightarrow$ $b^2 - 4ac = -0 \quad \Rightarrow \quad b^2 - 4ac = 0$

So, roots are repeated.

Therefore, the roots of the equation are imaginary unless $ad = bc$, in which case the roots are repeated.

Q8. Show that the roots of the equation $(ax+c)^2 = 4bx$ will be equal, if $b = ac$.

Solution: $(ax+c)^2 = 4bx$

$\Rightarrow$ $a^2x^2 + c^2 + 2acx - 4bx = 0$

$\Rightarrow$ $a^2x^2 + (2ac - 4b)x + c^2 = 0$

$a = a^2, \quad b = 2ac - 4b, \quad c = c^2$

Now, for the roots to be equal, the discriminant must be equal to zero.

Disc $= b^2 - 4ac = 0$

$\Rightarrow$ $(2ac - 4b)^2 - 4(a^2)(c^2) = 0$

$\Rightarrow$ $4a^2c^2 + 16b^2 - 16abc - 4a^2c^2 = 0$

$\Rightarrow$ $16b^2 - 16abc = 0 \quad \Rightarrow \quad 16b(b - ac) = 0$

$\Rightarrow$ $b - ac = 0 \quad \Rightarrow \quad b = ac$

Hence, proved.

Q9. <u>Show that the roots of the following equations are real.</u>

(i) $mx^2 - 2mx + m - 1 = 0$

Solution: $mx^2 - 2mx + m - 1 = 0$

$a = m, b = -2m, c = m - 1$

$\Rightarrow$ Disc $= b^2 - 4ac$

$= (-2m)^2 - 4(m)(m-1)$

$= 4m^2 - 4m^2 + 4m = 4m > 0$

$= b^2 - 4ac > 0$

For the roots to be real, the discriminant must be greater than or equal to zero.

So, roots are real. Hence, proved.

(ii) $bx^2 + ax + a - b = 0$

Solution: $bx^2 + ax + a - b = 0$

$a = b,\quad b = a,\quad c = a - b$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (a)^2 - 4(b)(a - b)$

$= a^2 - 4ab + 4b^2$

$= (a)^2 - 2(a)(2b) + (2b)^2$

$= (a - 2b)^2 > 0$

$= b^2 - 4ac > 0$

For the roots to be real, the discriminant must be greater than or equal to zero.

So, roots are real. Hence, proved.

Q10. Show that the roots of the following equation are real.

➢ $(a + b)x^2 - ax - b = 0$

Solution: $(a + b)x^2 - ax - b = 0$

$a = a + b,\quad b = -a,\quad c = -b$

$\Rightarrow$ $\text{Disc} = b^2 - 4ac$

$= (-a)^2 - 4(a + b)(-b)$

$= a^2 + 4b(a + b)$

$= a^2 + 4ab + 4b^2$

$= a^2 + 2(a)(2b) + (2b)^2$

$\Rightarrow$ $\text{Disc} = (a + 2b)^2 > 0$

Since $(a + 2b)^2$ is a square of a real number, it is always non-negative. Therefore, we have:

Since $\text{Disc} = b^2 - 4ac > 0$. So, roots are real.

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