Exercise 2.2

Q1. Reduce the following equations to quadratic form using suitable substitution.

i. $ax^4 + bx^2 + c = 0$

Solution:

Let's use the substitution $y = x^2$.

Then the given equation $ax^4 + bx^2 + c = 0$ can be rewritten in terms of y.

$\Rightarrow$ $ay^2 + by + c = 0$

ii. $9x^6 - 3x^3 + 7 = 0$

Solution:

Let's use the substitution $y = x^3$.

Then the given equation $9x^6 - 3x^3 + 7 = 0$ can be rewritten in terms of y.

$\Rightarrow$ $9y^2 - 3y + 7 = 0$

iii. $3x + \frac{4}{6x-2} = -1$

Solution:

Let's first multiply through by $6x - 2$ to clear the fraction:

$\Rightarrow$ $3x(6x-2) + 4 = -1(6x-2)$

Expanding both sides gives:

$\Rightarrow$ $18x^2 - 6x + 4 = -6x + 2$

Now move all terms to one side of the equation:

$\Rightarrow$ $18x^2 - 6x + 4 + 6x - 2 = 0$

$\Rightarrow$ $18x^2 + 2 = 0$ $\Rightarrow$ $9x^2 + 1 = 0$ $\Rightarrow$ $9x^2 + 0x + 1 = 0$

iv. $(x+1)^2 + \frac{3}{(x+1)^2} = 4$

Solution:

Let's use the substitution $y = x + 1$.

Then the given equation $(x+1)^2 + \frac{3}{(x+1)^2} = 4$ can be rewritten in terms of y.

$\Rightarrow$ $y^2 + \frac{3}{y^2} = 4$

To clear the fraction, we multiply through by y^2.

$\Rightarrow$ $y^4 + 3 = 4y^2$

Now, let's move all terms to one side of the equation to form a quadratic equation in y^2.

$\Rightarrow \quad y^4 - 4y^2 + 3 = 0$

Let's make another substitution, $z = y^2$, to make it a standard quadratic equation.

$\Rightarrow \quad z^2 - 4z + 3 = 0$

v. $3\left(x^2 + \frac{1}{x^2}\right) + 8\left(x + \frac{1}{x}\right) + 11 = 0$

Solution:

Let's use the substitution $y = x + \frac{1}{x}$

Notice that $y^2 = \left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$, so $x^2 + \frac{1}{x^2} = y^2 - 2$.

Substitute into the original equation:

$\Rightarrow \quad 3\left(x^2 + \frac{1}{x^2}\right) + 8\left(x + \frac{1}{x}\right) + 11 = 0$

$\Rightarrow \quad 3(y^2 - 2) + 8y + 11 = 0 \quad \Rightarrow \quad 3y^2 - 6 + 8y + 11 = 0$

$\Rightarrow \quad 3y^2 + 8y + 5 = 0$

vi. $3x^4 + 7x^3 + 5x^2 - 7x + 3 = 0$

Solution: $3x^4 + 7x^3 + 5x^2 - 7x + 3 = 0$ (reciprocal equation)

Dividing both sides by x^2 we get:

$\Rightarrow \quad 3x^2 + 7x + 5 - \frac{7}{x} + \frac{3}{x^2} = 0$

By rearranging, we get:

$\Rightarrow \quad \left(3x^2 + \frac{3}{x^2}\right) + \left(7x - \frac{7}{x}\right) + 5 = 0$

$\Rightarrow \quad 3\left(x^2 + \frac{1}{x^2}\right) + 7\left(x - \frac{1}{x}\right) + 5 = 0 \quad$ .... (i)

Let, $x - \frac{1}{x} = z$; (By squaring on both sides)

$\Rightarrow \quad x^2 + \frac{1}{x^2} - 2 = z^2 \quad \Rightarrow \quad x^2 + \frac{1}{x^2} = z^2 + 2$

Then equation (i) becomes,

$\Rightarrow \quad 3(z^2 + 2) + 7z + 5 = 0 \quad \Rightarrow \quad 3z^2 + 6 + 7z + 5 = 0$

$\Rightarrow \quad 3z^2 + 7z + 11 = 0$

vii. $ak^{2x} + bk^x + c = 0$

Solution: $ak^{2x} + bk^x + c = 0$

We can use the substitution $z = k^x$.

This transforms the exponential equation into a quadratic equation in terms of z.

Substituting $z = k^x$ into the equation, we get: $az^2 + bz + c = 0$

viii. $8 \times 4^x - 7 \times 2^x - 1 = 0$

Solution: $8 \times 4^x - 7 \times 2^x - 1 = 0$

We can use the substitution $y = 2^x$. Since $4^x = (2^x)^2 = y^2$, the equation becomes a quadratic in terms of y.

Substituting $y = 2^x$ into the equation, we get: $8y^2 - 7y - 1 = 0$

ix. $(x-1)(x-2)(x+3)(x-6) = 6$

Solution: $(x-1)(x-2)(x+3)(x-6) = 6$

$\Rightarrow$ $[(x-1)(x-2)][(x+3)(x-6)] = 6$

$\Rightarrow$ $(x^2 - 3x + 2)(x^2 - 3x - 18) = 6$ ……… (1)

Let, $x^2 - 3x = y$

Then equation (1) becomes,

$\Rightarrow$ $(y+2)(y-18) = 6$ $\Rightarrow$ $y^2 - 16y - 36 = 6$

$\Rightarrow$ $y^2 - 16y - 36 - 6 = 0$ $\Rightarrow$ $y^2 - 16y - 42 = 0$

x. $(2x-1)(2x-7)(x-3)(x-1) = 8$

Solution: $(2x-1)(2x-7)(x-3)(x-1) = 8$

$\Rightarrow$ $(4x^2 - 14x - 2x + 7)(x^2 - 4x + 3) = 8$

$\Rightarrow$ $(4x^2 - 16x + 7)(x^2 - 4x + 3) = 8$

$\Rightarrow$ $[4(x^2 - 4x) + 7](x^2 - 4x + 3) = 8$ ……… (1)

Let, $x^2 - 4x = z$

Then equation (1) becomes,

$\Rightarrow$ $(4z + 7)(z + 3) = 8$

$\Rightarrow$ $4z^2 + 12z + 7z + 21 - 8 = 0$

$\Rightarrow$ $4z^2 + 19z + 13 = 0$

Q2. <u>Solve the following equations by reducing them to quadratic form.</u>

i. $x^4 - 20x^2 + 64 = 0$

Solution:

We can reduce it to quadratic form by making the substitution $y = x^2$. Then $y^2 = (x^2)^2 = x^4$, and the equation becomes:

$\Rightarrow$ $y^2 - 20y + 64 = 0$

Now we factor the quadratic equation.

$\Rightarrow$ $(y-16)(y-4) = 0$

Setting each factor equal to zero gives us the solutions for y.

$\Rightarrow$ $y - 16 = 0$ $\Rightarrow$ $y = 16$

$\Rightarrow$ $y - 4 = 0$ $\Rightarrow$ $y = 4$

Recall that $y = x^2$, so we now solve for x.

$\Rightarrow$ $x^2 = 16$ $\Rightarrow$ $x = \pm\sqrt{16} = \pm 4$

$\Rightarrow$ $x^2 = 4$ $\Rightarrow$ $x = \pm\sqrt{4} = \pm 2$

$\therefore$ **Solution set** $= \{\pm 2, \pm 4\}$

ii. $x^4 + 16x^2 - 225 = 0$

Solution:

Let, $y = x^2$ so that $y^2 = x^4$. The equation becomes:

$\Rightarrow \quad y^2 + 16y - 225 = 0$

Now we can use the quadratic formula to solve for y.

$$\Rightarrow \quad y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $a = 1$, $b = 16$, and $c = -225$. Substituting these values gives:

$$\Rightarrow \quad y = \frac{-16 \pm \sqrt{16^2 - 4(1)(-225)}}{2(1)}$$

$$\Rightarrow \quad y = \frac{-16 \pm \sqrt{256 + 900}}{2} \quad \Rightarrow \quad y = \frac{-16 \pm \sqrt{1156}}{2}$$

$$\Rightarrow \quad y = \frac{-16 \pm 34}{2}$$

We have two possible solutions for y.

$$\Rightarrow \quad y_1 = \frac{-16 + 34}{2} = \frac{18}{2} = 9 \quad \Rightarrow \quad y_2 = \frac{-16 - 34}{2} = \frac{-50}{2} = -25$$

Recall that $y = x^2$, so we now solve for x.

$\Rightarrow \quad x^2 = 9 \quad ; \quad x^2 = -25$

$\Rightarrow \quad x = \pm 3 \quad ; \quad x = \pm\sqrt{-25} = \pm 5i$

$\therefore$ **Solution set** $= \{\pm 3, \pm 5i\}$

iii. $x^{\frac{2}{5}} + 5x^{\frac{1}{5}} + 6 = 0$

Solution:

To solve the equation $x^{\frac{2}{5}} + 5x^{\frac{1}{5}} + 6 = 0$, we can reduce it to quadratic form by letting $y = x^{\frac{1}{5}}$.

Then, $y^2 = \left(x^{\frac{1}{5}}\right)^2 = x^{\frac{2}{5}}$, and the equation becomes: $y^2 + 5y + 6 = 0$

We can factor this quadratic equation: $(y + 2)(y + 3) = 0$

Setting each factor equal to zero gives us the solutions for y:

$\Rightarrow \quad y + 2 = 0 \quad \Rightarrow \quad y = -2$

$\Rightarrow \quad y + 3 = 0 \quad \Rightarrow \quad y = -3$

Since $y = x^{\frac{1}{5}}$, we now solve for x:

$\Rightarrow \quad x^{\frac{1}{5}} = -2 \quad \Rightarrow \quad x = (-2)^5 = -32$

$\Rightarrow \quad x^{\frac{1}{5}} = -3 \quad \Rightarrow \quad x = (-3)^5 = -243$

$\therefore$ **Solution set** $= \{-32, -243\}$

iv. $3x^2 + \frac{4}{x^2} = 7$

Solution:

To solve the equation $3x^2 + \frac{4}{x^2} = 7$, let's first multiply through by x^2 to get rid of the fraction:

$\Rightarrow \quad x^2(3x^2) + x^2\left(\frac{4}{x^2}\right) = x^2(7)$

$\Rightarrow \quad 3x^4 + 4 = 7x^2 \quad \Rightarrow \quad 3x^4 - 7x^2 + 4 = 0$

Let $y = x^2$, then the equation becomes:

$\Rightarrow \quad 3y^2 - 7y + 4 = 0$

We can now factor the quadratic equation:

$\Rightarrow \quad (3y - 4)(y - 1) = 0$

Setting each factor equal to zero gives us the solutions for y:

$\Rightarrow \quad 3y - 4 = 0 \quad \Rightarrow \quad y = \frac{4}{3}$

$\Rightarrow \quad y - 1 = 0 \quad \Rightarrow \quad y = 1$

Recall that $y = x^2$, so we solve for x:

$\Rightarrow \quad x^2 = \frac{4}{3} \quad \Rightarrow \quad x = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$

$\Rightarrow \quad x^2 = 1 \quad \Rightarrow \quad x = \pm\sqrt{1} = \pm 1$

$\therefore$ **Solution set** $= \left\{\pm 1, \ \pm\frac{2}{\sqrt{3}}\right\}$

v. $5(x+1) + \frac{3}{x+1} = 8$

Solution:

To solve the equation $5(x+1) + \frac{3}{x+1} = 8$, let's first multiply through by $(x+1)$ to get rid of the fraction:

$\Rightarrow \quad (x+1)[5(x+1)] + (x+1)\left(\frac{3}{x+1}\right) = (x+1)(8)$

$\Rightarrow \quad 5(x+1)^2 + 3 = 8(x+1)$

Expanding the terms, we get:

$\Rightarrow \quad 5(x^2 + 2x + 1) + 3 = 8x + 8 \quad \Rightarrow \quad 5x^2 + 10x + 5 + 3 = 8x + 8$

Now, we rearrange the equation to set it to zero:

$\Rightarrow \quad 5x^2 + 10x + 8 = 8x + 8 \quad \Rightarrow \quad 5x^2 + 10x - 8x + 8 - 8 = 0$

$\Rightarrow \quad 5x^2 + 2x = 0$

We can now factor out an x. $\Rightarrow \quad x(5x + 2) = 0$

Setting each factor equal to zero gives us the solutions for x:

$\Rightarrow \quad x = 0$

$\Rightarrow \quad 5x + 2 = 0 \quad \Rightarrow \quad x = -\frac{2}{5}$

$\therefore$ **Solution set** $= \left\{0, -\frac{2}{5}\right\}$

vi. $5x^2 + \frac{36}{5x^2+4} = 16$

Solution:

To solve the equation $5x^2 + \frac{36}{5x^2+4} = 16$, let's first isolate the fraction:

$$\Rightarrow \quad 5x^2 + \frac{36}{5x^2+4} = 16 \quad \Rightarrow \quad \frac{36}{5x^2+4} = 16 - 5x^2$$

Now, we can multiply both sides by $5x^2 + 4$ to eliminate the denominator:

$$\Rightarrow \quad 36 = (16 - 5x^2)(5x^2 + 4)$$

Expanding the right hand side, we get:

$$\Rightarrow \quad 36 = 80x^2 - 25x^4 + 64 - 20x^2$$

Combine like terms and bring all terms to one side of the equation:

$$\Rightarrow \quad 25x^4 - 60x^2 - 28 = 0$$

We can reduce it to quadratic form by making the substitution $y = x^2$.
Then $y^2 = (x^2)^2 = x^4$, and the equation becomes:

$$\Rightarrow \quad 25y^2 - 60y - 28 = 0$$

$$\Rightarrow \quad 25y^2 - 70y + 10y - 28 = 0$$

Now we factor the quadratic equation:

$$\Rightarrow \quad 5y(5y - 14) + 2(5y - 14) = 0$$

$$\Rightarrow \quad (5y - 14)(5y + 2) = 0$$

Setting each factor equal to zero gives us the solutions for y.

$$\Rightarrow \quad 5y - 14 = 0 \quad \Rightarrow \quad y = \frac{14}{5}$$

$$\Rightarrow \quad 5y + 2 = 0 \quad \Rightarrow \quad 5y = -2 \quad \Rightarrow \quad y = \frac{-2}{5}$$

Recall that $y = x^2$, so we solve for x.

$$\Rightarrow \quad x^2 = \frac{14}{5} \quad \Rightarrow \quad x = \pm\sqrt{\frac{14}{5}} \quad \Rightarrow \quad x^2 = \frac{-2}{5} \quad \Rightarrow \quad x = \pm\sqrt{\frac{-2}{5}}$$

$$\therefore \quad \textbf{Solution set} = \left\{\pm\sqrt{\frac{14}{5}}, \pm\sqrt{\frac{-2}{5}}\right\}$$

vii. $2x^4 - x^3 - 6x^2 - x + 2 = 0$

Solution: $2x^4 - x^3 - 6x^2 - x + 2 = 0$

Dividing both sides by 'x^2'.

$$\Rightarrow \quad \frac{2x^4}{x^2} - \frac{x^3}{x^2} - \frac{6x^2}{x^2} - \frac{x}{x^2} + \frac{2}{x^2} = \frac{0}{x^2} \quad \Rightarrow \quad 2x^2 - x - 6 - \frac{1}{x} + \frac{2}{x^2} = 0$$

$$\Rightarrow \quad 2\left(x^2 + \frac{1}{x^2}\right) - 1\left(x + \frac{1}{x}\right) - 6 = 0$$

Let, $y = x + \frac{1}{x}$

$$\Rightarrow \quad y^2 = \left(x + \frac{1}{x}\right)^2 \quad ; \quad (a+b)^2 = a^2 + b^2 + 2ab$$

$\Rightarrow \quad y^2 = x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) \quad \Rightarrow \quad y^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow \quad y^2 - 2 = x^2 + \frac{1}{x^2} \quad \Rightarrow \quad 2(y^2 - 2) - 1(y) - 6 = 0$

$\Rightarrow \quad 2y^2 - 4 - y - 6 = 0 \quad \Rightarrow \quad 2y^2 - y - 10 = 0$

$\Rightarrow \quad 2y^2 - 5y + 4y - 10 = 0 \quad \Rightarrow \quad y(2y - 5) + 2(2y - 5) = 0$

$\Rightarrow \quad (2y - 5)(y + 2) = 0$

$\Rightarrow \quad 2y - 5 = 0$

$\Rightarrow \quad 2y = 5$

$\Rightarrow \quad y = \frac{5}{2}$

$\Rightarrow \quad y + 2 = 0$

$\Rightarrow \quad y = -2$

Now, putting values in $y = x + \frac{1}{x}$.

$\Rightarrow \quad x + \frac{1}{x} = \frac{5}{2}$

$\Rightarrow \quad \frac{x^2 + 1}{x} = \frac{5}{2}$

$\Rightarrow \quad 2x^2 + 2 = 5x$

$\Rightarrow \quad 2x^2 - 5x + 2 = 0$

$\Rightarrow \quad 2x^2 - 4x - x + 2 = 0$

$\Rightarrow \quad 2x(x - 2) - 1(x - 2) = 0$

$\Rightarrow \quad (2x - 1)(x - 2) = 0$

$\Rightarrow \quad 2x = 1 \quad ; \quad x = 2$

$\Rightarrow \quad x = \frac{1}{2} \quad ; \quad x = 2$

$\Rightarrow \quad x + \frac{1}{x} = -2$

$\Rightarrow \quad \frac{x^2 + 1}{x} = -2$

$\Rightarrow \quad x^2 + 1 = -2x$

$\Rightarrow \quad x^2 + 2x + 1 = 0$

$\Rightarrow \quad x^2 + x + x + 1 = 0$

$\Rightarrow \quad x(x + 1) + 1(x + 1) = 0$

$\Rightarrow \quad (x + 1)(x + 1) = 0$

$\Rightarrow \quad \sqrt{(x + 1)^2} = \sqrt{0}$

$\Rightarrow \quad x + 1 = 0 \quad \Rightarrow \quad x = -1$

$\therefore$ **Solution set** $= \left\{-1,\ 2,\ \frac{1}{2}\right\}$

viii. $12x^4 + 11x^3 - 146x^2 + 11x + 12 = 0$

Solution: $12x^4 + 11x^3 - 146x^2 + 11x + 12 = 0$

Dividing both sides by 'x^2'.

$\Rightarrow \quad \frac{12x^4}{x^2} + \frac{11x^3}{x^2} - \frac{146x^2}{x^2} + \frac{11x}{x^2} + \frac{12}{x^2} = \frac{0}{x^2}$

$\Rightarrow \quad 12x^2 + 11x - 146 + \frac{11}{x} + \frac{12}{x^2} = 0$

$\Rightarrow \quad 12\left(x^2 + \frac{1}{x^2}\right) + 11\left(x + \frac{1}{x}\right) - 146 = 0$

Let, $\quad x + \frac{1}{x} = y$

$\Rightarrow \quad \left(x + \frac{1}{x}\right)^2 = y^2 \quad ; \quad (a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow \quad x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = y^2 \qquad \Rightarrow \quad x^2 + \frac{1}{x^2} = y^2 - 2$

$\Rightarrow \quad 12(y^2 - 2) + 11(y) - 146 = 0 \qquad \Rightarrow \quad 12y^2 - 24 + 11y - 146 = 0$

$\Rightarrow \quad 12y^2 + 11y - 170 = 0 \qquad \Rightarrow \quad 12y^2 - 40y + 51y - 170 = 0$

$\Rightarrow \quad 4y(3y - 10) + 17(3y - 10) = 0$

$\Rightarrow \quad (3y - 10)(4y + 17) = 0$

$\Rightarrow \quad 3y - 10 = 0 \qquad | \qquad \Rightarrow \quad 4y + 17 = 0$

$\Rightarrow \quad 3y = 10 \qquad | \qquad \Rightarrow \quad 4y = -17$

$\Rightarrow \quad y = \frac{10}{3} \qquad | \qquad \Rightarrow \quad y = \frac{-17}{4}$

Now, putting values in $y = x + \frac{1}{x}$.

$\Rightarrow \quad x + \frac{1}{x} = \frac{10}{3} \qquad | \qquad \Rightarrow \quad x + \frac{1}{x} = -\frac{17}{4}$

$\Rightarrow \quad \frac{x^2 + 1}{x} = \frac{10}{3} \qquad | \qquad \Rightarrow \quad \frac{x^2 + 1}{x} = -\frac{17}{4}$

$\Rightarrow \quad 3x^2 + 3 = 10x \qquad | \qquad \Rightarrow \quad 4x^2 + 4 = -17x$

$\Rightarrow \quad 3x^2 - 10x + 3 = 0 \qquad | \qquad \Rightarrow \quad 4x^2 + 17x + 4 = 0$

$\Rightarrow \quad 3x^2 - 9x - x + 3 = 0 \qquad | \qquad \Rightarrow \quad 4x^2 + 16x + x + 4 = 0$

$\Rightarrow \quad 3x(x - 3) - 1(x - 3) = 0 \qquad | \qquad \Rightarrow \quad 4x(x + 4) + 1(x + 4) = 0$

$\Rightarrow \quad (x - 3)(3x - 1) = 0 \qquad | \qquad \Rightarrow \quad (x + 4)(4x + 1) = 0$

$\Rightarrow \quad x - 3 = 0 \quad ; \quad 3x = 1 \qquad | \qquad \Rightarrow \quad x + 4 = 0 \quad ; \quad 4x = -1$

$\Rightarrow \quad x = 3 \quad ; \quad x = \frac{1}{3} \qquad | \qquad \Rightarrow \quad x = -4 \quad ; \quad x = \frac{-1}{4}$

$\therefore$ **Solution set** $= \left\{3, -4, \frac{1}{3}, -\frac{1}{4}\right\}$

ix. $4\left(x^2 + \frac{1}{x^2}\right) - \left(x - \frac{1}{x}\right) - 11 = 0$

Solution:

Let, $x - \frac{1}{x} = y$

$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 = y^2 \quad ; \quad (a - b)^2 = a^2 + b^2 - 2ab$

$\Rightarrow \quad x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right) = y^2 \qquad \Rightarrow \quad x^2 + \frac{1}{x^2} = y^2 + 2$

$\Rightarrow \quad 4(y^2 + 2) - y - 11 = 0 \qquad \Rightarrow \quad 4y^2 + 8 - y - 11 = 0$

$\Rightarrow \quad 4y^2 - y - 3 = 0 \qquad \Rightarrow \quad 4y^2 - 4y + 3y - 3 = 0$

$\Rightarrow \quad 4y(y - 1) + 3(y - 1) = 0$

$\Rightarrow \quad (y - 1)(4y + 3) = 0$

$\Rightarrow \quad y-1=0$

$\Rightarrow \quad y=1$

$\Rightarrow \quad 4y+3=0$

$\Rightarrow \quad 4y=-3$

$\Rightarrow \quad y=\frac{-3}{4}$

Now, putting values in $y = x - \frac{1}{x}$.

$\Rightarrow \quad x-\frac{1}{x}=1$

$\Rightarrow \quad \frac{x^2-1}{x}=1$

$\Rightarrow \quad x^2-1=x$

$\Rightarrow \quad x^2-x-1=0$

Here, $a=1, b=-1, c=-1$

$\Rightarrow \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\Rightarrow \quad x=\frac{1\pm\sqrt{1-4(1)(-1)}}{2(1)}$

$\Rightarrow \quad x=\frac{1\pm\sqrt{5}}{2}$

$\Rightarrow \quad x-\frac{1}{x}=-\frac{3}{4}$

$\Rightarrow \quad \frac{x^2-1}{x}=-\frac{3}{4}$

$\Rightarrow \quad 4x^2-4=-3x$

$\Rightarrow \quad 4x^2+3x-4=0$

Here, $a=4, b=3, c=-4$

$\Rightarrow \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\Rightarrow \quad x=\frac{-3\pm\sqrt{9-4(4)(-4)}}{2(4)}$

$\Rightarrow \quad x=\frac{-3\pm\sqrt{9+64}}{8}$

$\Rightarrow \quad x=\frac{-3\pm\sqrt{73}}{8}$

$\therefore$ **Solution set** $=\left\{\frac{1\pm\sqrt{5}}{2}, \frac{-3\pm\sqrt{73}}{8}\right\}$

x. $2^{2x}-34\times 2^x+64=0$

Solution:

Let's introduce the substitution $y=2^x$. Then $y^2=(2^x)^2=2^{2x}$.

The equation becomes:

$\Rightarrow \quad y^2-34y+64=0 \quad \Rightarrow \quad y^2-32y-2y+64=0$

$\Rightarrow \quad y(y-32)-2(y-32)=0 \quad \Rightarrow \quad (y-32)(y-2)=0$

Setting each factor equal to zero gives us the solutions for y:

$\Rightarrow \quad y-32=0$ or $y-2=0$

$\Rightarrow \quad y=32$ or $y=2$

Now we need to substitute back $y=2^x$ to find the values of x:

- For $y=32$:

$\Rightarrow \quad 2^x=32 \quad \Rightarrow \quad 2^x=2^5 \quad \Rightarrow \quad x=5$

- For $y=2$:

$\Rightarrow \quad 2^x=2 \quad \Rightarrow \quad 2^x=2^1 \quad \Rightarrow \quad x=1$

Combining both solutions, we have:

$\Rightarrow \quad x=5$ or $x=1$

$\therefore$ **Solution set** $=\{1,5\}$

xi. $3^{2x} - 12 \times 3^x + 27 = 0$

Solution:

Let's introduce the substitution $y = 3^x$. Then $y^2 = (3^x)^2 = 3^{2x}$.

The equation becomes:

$\Rightarrow \quad y^2 - 12y + 27 = 0$

$\Rightarrow \quad y^2 - 9y - 3y + 27 = 0$

$\Rightarrow \quad y(y-9) - 3(y-9) = 0$

$\Rightarrow \quad (y-9)(y-3) = 0$

Setting each factor equal to zero gives us the solutions for y:

$\Rightarrow \quad y - 9 = 0 \quad$ or $\quad y - 3 = 0$

$\Rightarrow \quad y = 9 \quad$ or $\quad y = 3$

Now we need to substitute back $y = 3^x$ to find the values of x:

- For $y = 9$:

$\Rightarrow \quad 3^x = 9 \quad \Rightarrow \quad 3^x = 3^2 \quad \Rightarrow \quad x = 2$

- For $y = 3$:

$\Rightarrow \quad 3^x = 3 \quad \Rightarrow \quad 3^x = 3^1 \quad \Rightarrow \quad x = 1$

Combining both solutions, we have:

$\Rightarrow \quad x = 2 \quad$ or $\quad x = 1$

$\therefore \quad$ **Solution set** $= \{1, 2\}$

xii. $5^{2x} - 150 \times 5^x + 3125 = 0$

Solution:

Let's introduce the substitution $y = 5^x$. Then $y^2 = (5^x)^2 = 5^{2x}$.

The equation becomes:

$\Rightarrow \quad y^2 - 150y + 3125 = 0$

$\Rightarrow \quad y^2 - 125y - 25y + 3125 = 0$

$\Rightarrow \quad y(y-125) - 25(y-125) = 0$

$\Rightarrow \quad (y-125)(y-25) = 0$

Setting each factor equal to zero gives us the solutions for y:

$\Rightarrow \quad y - 125 = 0 \quad$ or $\quad y - 25 = 0$

$\Rightarrow \quad y = 125 \quad$ or $\quad y = 25$

Now we need to substitute back $y = 5^x$ to find the values of x:

- For $y = 125$:

$\Rightarrow \quad 5^x = 125 \quad \Rightarrow \quad 5^x = 5^3 \quad \Rightarrow \quad x = 3$

- For $y = 25$:

$\Rightarrow \quad 5^x = 25 \quad \Rightarrow \quad 5^x = 5^2 \quad \Rightarrow \quad x = 2$

Combining both solutions, we have:

$\Rightarrow \quad x = 3 \quad$ or $\quad x = 2$

$\therefore \quad$ ***Solution set*** $= \{2, 3\}$

xiii. $(x+2)(x-3)(x+10)(x+5) = -396$

Solution:

$\Rightarrow \quad (x+2)(x+5)(x-3)(x+10) = -396 \quad ; \quad$ **(By rearranging)**

$\Rightarrow \quad (x^2 + 7x + 10)(x^2 + 7x - 30) = -396$

Let, $x^2 + 7x = y$

$\Rightarrow (y+10)(y-30) = -396 \Rightarrow y^2 - 20y - 300 + 396 = 0$

$\Rightarrow y^2 - 20y + 96 = 0 \Rightarrow y^2 - 8y - 12y + 96 = 0$

$\Rightarrow y(y-8) - 12(y-8) = 0$

$\Rightarrow (y-8)(y-12) = 0$

$\Rightarrow y - 8 = 0$ | $\Rightarrow y - 12 = 0$

$\Rightarrow y = 8$ | $\Rightarrow y = 12$

Now, putting values in $y = x^2 + 7x$.

$\Rightarrow x^2 + 7x = 8$

$\Rightarrow x^2 + 7x - 8 = 0$

$\Rightarrow x^2 - 1x + 8x - 8 = 0$

$\Rightarrow x(x-1) + 8(x-1) = 0$

$\Rightarrow (x-1)(x+8) = 0$

$\Rightarrow x - 1 = 0 \; ; \; x + 8 = 0$

$\Rightarrow x = 1 \; ; \; x = -8$

$\Rightarrow x^2 + 7x = 12$

$\Rightarrow x^2 + 7x - 12 = 0$

Here, $a = 1, b = 7, c = -12$

$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-12)}}{2(1)}$

$\Rightarrow x = \frac{-7 \pm \sqrt{49 + 48}}{2}$

$\Rightarrow x = \frac{-7 \pm \sqrt{97}}{2}$

$\therefore$ Solution set $= \left\{1, -8, \frac{-7 \pm \sqrt{97}}{2}\right\}$

xiv. $x(x-1)(x+2)(x+3) = 40$

Solution:

$x(x+2)(x-1)(x+3) = 40$; (By rearranging)

$(x^2 + 2x)(x^2 + 3x - x - 3) = 40$

$(x^2 + 2x)(x^2 + 2x - 3) = 40$

Let, $x^2 + 2x = y$

$\Rightarrow y(y-3) = 40 \Rightarrow y^2 - 3y = 40$

$\Rightarrow y^2 - 3y - 40 = 0 \Rightarrow y^2 - 8y + 5y - 40 = 0$

$\Rightarrow y(y-8) + 5(y-8) = 0$

$\Rightarrow (y+5)(y-8) = 0$

$\Rightarrow y + 5 = 0$ | $\Rightarrow y - 8 = 0$

$\Rightarrow y = -5$ | $\Rightarrow y = 8$

Now, putting values in $y = x^2 + 2x$.

$\Rightarrow x^2 + 2x = -5$

$\Rightarrow x^2 + 2x + 5 = 0$

Here, $a = 1, b = 2, c = 5$

$\Rightarrow x^2 + 2x = 8$

$\Rightarrow x^2 + 2x - 8 = 0$

$\Rightarrow x^2 + 4x - 2x - 8 = 0$

$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(5)}}{2(1)}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{4 - 20}}{2}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{-16}}{2}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{(-4 \times 4)}}{2}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{(-4)^2}}{2} = \frac{-2 \pm 4i}{2}$

$\Rightarrow \quad x = \frac{-2(-1 \pm 2i)}{2} = -1 \pm 2i$

$\Rightarrow \quad x(x+4) - 2(x+4) = 0$

$\Rightarrow \quad (x-2)(x+4) = 0$

$\Rightarrow \quad x - 2 = 0 \;;\; x + 4 = 0$

$\Rightarrow \quad x = 2 \;;\; x = -4$

$\therefore$ Solution set $= \{2, -4, -1 \pm 2i\}$

xv. $(x-2)(x-6)(x+4)(x+8) + 256 = 0$

Solution:

$(x-2)(x+4)(x-6)(x+8) + 256 = 0$; (By rearranging)

$(x^2 + 2x - 8)(x^2 + 2x - 48) + 256 = 0$

Let, $x^2 + 2x = y$

$\Rightarrow \quad (y-8)(y-48) + 256 = 0 \quad \Rightarrow \quad y^2 - 56y + 384 + 256 = 0$

$\Rightarrow \quad y^2 - 56y + 640 = 0 \quad \Rightarrow \quad y^2 - 16y - 40y + 640 = 0$

$\Rightarrow \quad y(y-16) - 40(y-16) = 0$

$\Rightarrow \quad (y-16)(y-40) = 0$

$\Rightarrow \quad y - 16 = 0$

$\Rightarrow \quad y = 16$

$\Rightarrow \quad y - 40 = 0$

$\Rightarrow \quad y = 40$

Now, putting values in $y = x^2 + 2x$.

$\Rightarrow \quad x^2 + 2x = 16$

$\Rightarrow \quad x^2 + 2x - 16 = 0$

Here, $a = 1,\ b = 2,\ c = -16$

$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-16)}}{2(1)}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{4 + 64}}{2}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{68}}{2}$

$\Rightarrow \quad x^2 + 2x = 40$

$\Rightarrow \quad x^2 + 2x - 40 = 0$

Here, $a = 1,\ b = 2,\ c = -40$

$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-40)}}{2(1)}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{4 + 160}}{2}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{164}}{2}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{(2)^2 \times 17}}{2}$

$\Rightarrow \quad x = \frac{-2 \pm 2\sqrt{17}}{2}$

$\Rightarrow \quad x = \frac{2(-1 \pm \sqrt{17})}{2}$

$\Rightarrow \quad x = -1 \pm \sqrt{17}$

$\Rightarrow \quad x = \frac{-2 \pm \sqrt{(2)^2 \times 41}}{2}$

$\Rightarrow \quad x = \frac{-2 \pm 2\sqrt{41}}{2}$

$\Rightarrow \quad x = \frac{2(-1 \pm \sqrt{41})}{2}$

$\Rightarrow \quad x = -1 \pm \sqrt{41}$

$\therefore$ Solution set $= \{-1 \pm \sqrt{17},\ -1 \pm \sqrt{41}\}$

Q3. Solve the following equations by factoring without using substitution.

i. $2x^4 - 3x^2 + 1 = 0$

Solution:

$\Rightarrow \quad 2x^4 - 3x^2 + 1 = 0$

$\Rightarrow \quad 2x^4 - 2x^2 - x^2 + 1 = 0$

$\Rightarrow \quad 2x^2(x^2 - 1) - 1(x^2 - 1) = 0$

$\Rightarrow \quad (2x^2 - 1)(x^2 - 1) = 0$

$\Rightarrow \quad 2x^2 - 1 = 0$

$\Rightarrow \quad 2x^2 = 1$

$\Rightarrow \quad x^2 = \frac{1}{2}$

$\Rightarrow \quad \sqrt{x^2} = \sqrt{\frac{1}{2}}$

$\Rightarrow \quad x = \pm \frac{1}{\sqrt{2}}$

$\Rightarrow \quad x^2 - 1 = 0$

$\Rightarrow \quad x^2 = 1$

$\Rightarrow \quad \sqrt{x^2} = \sqrt{1}$

$\Rightarrow \quad x = \pm 1$

$\therefore$ Solution set $= \left\{\pm 1,\ \pm \frac{1}{\sqrt{2}}\right\}$

ii. $8x^6 - 7x^3 - 1 = 0$ **(Find only real roots.)**

Solution:

$\Rightarrow \quad 8x^6 - 7x^3 - 1 = 0$

$\Rightarrow \quad 8x^6 - 8x^3 + x^3 - 1 = 0$

$\Rightarrow \quad 8x^3(x^3 - 1) + 1(x^3 - 1) = 0$

$\Rightarrow \quad (8x^3 + 1)(x^3 - 1) = 0$

$\Rightarrow \quad 8x^3 + 1 = 0$

$\Rightarrow \quad (2x)^3 + (1)^3 = 0$

$\Rightarrow \quad (2x + 1)(4x^2 - 2x + 1) = 0$

$\Rightarrow \quad 2x + 1 = 0 \quad \Rightarrow \quad 2x = -1$

$\Rightarrow \quad x = \frac{-1}{2}$

$\Rightarrow \quad x^3 - 1 = 0$

$\Rightarrow \quad (x)^3 - (1)^3 = 0$

$\Rightarrow \quad (x - 1)(x^2 + x + 1) = 0$

$\Rightarrow \quad x - 1 = 0$

$\Rightarrow \quad x = 1$

$\Rightarrow$ Now, $4x^2 - 2x + 1 = 0$

$\Rightarrow$ Here, $a = 4,\ b = -2,\ c = 1$

$\Rightarrow$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow$ $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(1)}}{2(4)}$

$\Rightarrow$ $x = \dfrac{2 \pm \sqrt{4 - 16}}{8}$

$\Rightarrow$ $x = \dfrac{2 \pm \sqrt{-12}}{8}$

$\Rightarrow$ $x = \dfrac{2 \pm \sqrt{12}\, i}{8}$ (Neglected)

$\Rightarrow$ Now, $x^2 + x + 1 = 0$

$\Rightarrow$ Here, $a = 1,\ b = 1,\ c = 1$

$\Rightarrow$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow$ $x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$\Rightarrow$ $x = \dfrac{-1 \pm \sqrt{1 - 4}}{2}$

$\Rightarrow$ $x = \dfrac{-1 \pm \sqrt{-3}}{2}$

$\Rightarrow$ $x = \dfrac{-1 \pm \sqrt{3}\, i}{2}$ (Neglected)

$\therefore$ **Solution set** $= \left\{1, -\dfrac{1}{2}\right\}$

iii. $x^2 + \dfrac{1}{x^2} = 2$

Solution:

$\Rightarrow$ $x^2 + \dfrac{1}{x^2} - 2 = 0$

$\Rightarrow$ $x^2 + \dfrac{1}{x^2} - 2(x)\left(\dfrac{1}{x}\right) = 0$; $[\because\ (a-b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow$ $\left(x - \dfrac{1}{x}\right)^2 = 0$ $\Rightarrow$ $\sqrt{\left(x - \dfrac{1}{x}\right)^2} = \sqrt{0}$

$\Rightarrow$ $x - \dfrac{1}{x} = 0$ $\Rightarrow$ $x = \dfrac{1}{x}$ $\Rightarrow$ $x^2 = 1$ $\Rightarrow$ $\sqrt{x^2} = \sqrt{1}$

$\Rightarrow$ $x = \pm 1$

$\therefore$ **Solution set** $= \{\pm 1\}$

iv. $4 \times 2^{2x} - 4 \times 2^x + 1 = 0$

Solution:

$\Rightarrow$ $4 \times 2^{2x} - 4 \times 2^x + 1 = 0$

$\Rightarrow$ $2 \times 2 \times 2^{2x} - 2 \times 2 \times 2^x + 1 = 0$

$\Rightarrow$ $2^2 \times 2^{2x} - 2 \times 2 \times 2^x + 1 = 0$

$\Rightarrow$ $(2 \times 2^x)^2 - 2(2 \times 2^x) + (1)^2 = 0$

$\Rightarrow$ $(2 \times 2^x - 1)^2 = 0$; $[\because\ (a-b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow$ $\sqrt{(2 \times 2^x - 1)^2} = \sqrt{0}$

$\Rightarrow$ $2 \times 2^x - 1 = 0$

$\Rightarrow$ $2 \times 2^x = 1$ $\Rightarrow$ $2^x = \dfrac{1}{2}$

$\Rightarrow$ $2^x = 2^{-1}$ $\Rightarrow$ $x = -1$

$\therefore$ **Solution set** $= \{-1\}$

(iii) Solve the Quadratic Equation:

First, rewrite the equation in standard form.

$\Rightarrow \quad x^2 - 7x + 6 = 0$

Now, factor the quadratic equation: $(x-6)(x-1) = 0$

So, the solutions for x are: $x = 6$ or $x = 1$

Substitute back $y-4$ for x: $y - 4 = 6$ or $y - 4 = 1$

Solve for y.

- For $y - 4 = 6$: $y = 6 + 4 = 10$
- For $y - 4 = 1$: $y = 1 + 4 = 5$

Therefore, the solutions for y are **10** and **5**.

Q6. Solve $y^3 = 125$.

Solution: $y^3 = 125$

We need to find the cube root of **125**.

$\Rightarrow \quad y = (125)^{1/3} \quad \Rightarrow \quad y^3 - 125 = 0 \quad \Rightarrow \quad (y)^3 - (5)^3 = 0$

Using, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$\Rightarrow \quad (y-5)(y^2 + 5y + 25) = 0$

Therefore, $y - 5 = 0$ gives $y = 5$ and $y^2 + 5y + 25 = 0$ gives:

Here, $a = 1,\ b = 5,\ c = 25$

$$\Rightarrow \quad y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad y = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(25)}}{2(1)} \quad \Rightarrow \quad y = \frac{-5 \pm \sqrt{25 - 100}}{2}$$

$$\Rightarrow \quad y = \frac{-5 \pm \sqrt{-75}}{2} \quad \Rightarrow \quad y = \frac{-5 \pm \sqrt{25 \times -3}}{2}$$

$$\Rightarrow \quad y = \frac{-5 \pm 5\sqrt{-3}}{2} \quad \Rightarrow \quad y = \frac{5(-1 \pm \sqrt{3 \times -1})}{2}$$

$$\Rightarrow \quad y = \frac{-5 \pm 5\sqrt{3}\,i}{2} \quad ; \quad (\because \quad i = \sqrt{-1})$$

$$\Rightarrow \quad y = \frac{-5 + 5\sqrt{3}\,i}{2} \quad \text{or} \quad y = \frac{-5 - 5\sqrt{3}\,i}{2}$$

$\therefore$ **Solution set** $= \left\{5, \dfrac{-5 + 5\sqrt{3}\,i}{2}, \dfrac{-5 - 5\sqrt{3}\,i}{2}\right\}$

❖ ❖ ❖ ❖ ❖