Exercise 2.1

Q1. Write the following quadratic equation in standard form:

i. $(x+2)(x-3)=5$

Solution: $(x+2)(x-3)=5$

Equation in standard form is given by $ax^2+bx+c=0.$

$\Rightarrow \quad (x+2)(x-3)=5$

$\Rightarrow \quad x^2-3x+2x-6=5 \quad \Rightarrow \quad x^2-x-6=5$

$\Rightarrow \quad x^2-x-6-5=0 \quad \Rightarrow \quad x^2-x-11=0$

ii. $(x-5)^2-(2x+4)^2=7$

Solution: $(x-5)^2-(2x+4)^2=7$

Equation in standard form is given by $ax^2+bx+c=0.$

$\Rightarrow \quad (x-5)^2-(2x+4)^2=7$

$\Rightarrow \quad (x^2-10x+25)-(4x^2+16x+16)=7$

$\Rightarrow \quad x^2-10x+25-4x^2-16x-16=7$

$\Rightarrow \quad -3x^2-26x+25-16=7 \quad \Rightarrow \quad -3x^2-26x+9=7$

$\Rightarrow \quad -3x^2-26x+9-7=0 \quad \Rightarrow \quad -3x^2-26x+2=0$

$\Rightarrow \quad 3x^2+26x-2=0$

iii. $x=x(x-1)$

Solution: $x=x(x-1)$

Equation in standard form is given by $ax^2+bx+c=0.$

$\Rightarrow \quad x=x(x-1) \quad \Rightarrow \quad x=x^2-x$

$\Rightarrow \quad 0=x^2-x-x \quad \Rightarrow \quad 0=x^2-2x$

$\Rightarrow \quad 0=x(x-2) \quad \Rightarrow \quad x^2-2x=0$

Q2. Solve the following equations by factoring method.

i. $(x-1)(x-4)=0$

Solution: $(x-1)(x-4)=0$

To solve the equation $(x-1)(x-4)=0$ by factorization, we set each factor equal to zero and solve for x:

$\Rightarrow \quad x-1=0 \quad ; \quad x-4=0$
$\Rightarrow \quad x=1 \quad ; \quad x=4$
$\therefore$ Solution set $=\{1,4\}$

ii. $x^2-2x+1=0$

Solution: $x^2-2x+1=0$
$\Rightarrow \quad x^2-x-x+1=0 \quad \Rightarrow \quad x(x-1)-1(x-1)=0$
$\Rightarrow \quad (x-1)(x-1)=0 \quad \Rightarrow \quad x-1=0 \quad \Rightarrow \quad x=1$
$\therefore$ Solution set $=\{1\}$

iii. $x^2-7x-8=0$

Solution: $x^2-7x-8=0$
$\Rightarrow \quad x^2+x-8x-8=0$
$\Rightarrow \quad x(x+1)-8(x+1)=0 \quad \Rightarrow \quad (x-8)(x+1)=0$
$\Rightarrow \quad x-8=0 \quad ; \quad x+1=0$
$\Rightarrow \quad x=8 \quad ; \quad x=-1$
$\therefore$ Solution set $=\{-1,8\}$

iv. $x^2-4x+4=(2x-7)^2$

Solution: $x^2-4x+4=(2x-7)^2$
$\Rightarrow \quad x^2-4x+4=(2x-7)(2x-7)$
$\Rightarrow \quad x^2-4x+4=4x^2-14x-14x+49$
$\Rightarrow \quad x^2-4x+4=4x^2-28x+49$
$\Rightarrow \quad 0=4x^2-28x+49-x^2+4x-4$
$\Rightarrow \quad 0=3x^2-24x+45 \quad \Rightarrow \quad x^2-8x+15=0$
$\Rightarrow \quad x^2-3x-5x+15=0 \quad \Rightarrow \quad x(x-3)-5(x-3)=0$
$\Rightarrow \quad (x-3)(x-5)=0$
$\Rightarrow \quad x-3=0 \quad ; \quad x-5=0$
$\Rightarrow \quad x=3 \quad ; \quad x=5$
$\therefore$ Solution set $=\{3,5\}$

v. $\left(2x+\frac{7}{4}\right)^2=\frac{48x^2+529}{16}$

Solution: $\left(2x+\frac{7}{4}\right)^2=\frac{48x^2+529}{16}$

$\Rightarrow \quad \left(\frac{8x}{4}+\frac{7}{4}\right)^2=\frac{48x^2+529}{16} \quad \Rightarrow \quad \frac{(8x+7)^2}{16}=\frac{48x^2+529}{16}$
$\Rightarrow \quad (8x+7)^2=48x^2+529$
$\Rightarrow \quad 64x^2+112x+49=48x^2+529$
$\Rightarrow \quad 64x^2+112x+49-48x^2-529=0$
$\Rightarrow \quad 16x^2+112x-480=0$

Divide the entire equation by 16 to simplify:

$\Rightarrow \quad x^2+7x-30=0 \quad \Rightarrow \quad x^2-3x+10x-30=0$
$\Rightarrow \quad x(x-3)+10(x-3)=0 \quad \Rightarrow \quad (x+10)(x-3)=0$

$\Rightarrow \quad x + 10 = 0 \quad ; \quad x - 3 = 0$

$\Rightarrow \quad x = -10 \quad ; \quad x = 3$

$\therefore$ Solution set $= \{3, -10\}$

Q3. Solve the following equations by completing the square method:

i. $x^2 + 4x - 32 = 0$

Solution: $x^2 + 4x = 32 \quad \ldots\ldots\ldots (1)$

Multiplying co-efficient of x with $\frac{1}{2}$. i.e., $\frac{1}{2}(4) = 2$

Now adding $(2)^2 = 4$ on both sides of the equation (1), we have:

$\Rightarrow \quad x^2 + 4x + 4 = 32 + 4 \quad \Rightarrow \quad x^2 + 4x + 4 = 36$

Now the left-hand side is a perfect square trinomial:

$\Rightarrow \quad (x + 2)^2 = 36$

Take the square root of both sides:

$\Rightarrow \quad x + 2 = \pm\sqrt{36} \quad \Rightarrow \quad x + 2 = \pm 6$

Finally, solve for x by subtracting 2 from both sides:

$\Rightarrow \quad x = -2 \pm 6$

The solutions to the equation are:

$\Rightarrow \quad x = -2 + 6 \quad ; \quad x = -2 - 6$

$\Rightarrow \quad x = 4 \quad ; \quad x = -8$

$\therefore$ Solution set $= \{4, -8\}$

ii. $x^2 + 8x = 0$

Solution: $x^2 = -8x \quad \ldots\ldots\ldots (1)$

Multiplying co-efficient of x with $\frac{1}{2}$. i.e., $\frac{1}{2}(8) = 4$

Now adding $(4)^2 = 16$ on both sides of the equation (1), we have:

$\Rightarrow \quad x^2 + 8x + 16 = 0 + 16$

$\Rightarrow \quad (x + 4)^2 = 16 \quad \Rightarrow \quad x + 4 = \pm\sqrt{16} \quad \Rightarrow \quad x + 4 = \pm 4$

Finally, solve for x.

$\Rightarrow \quad x + 4 = 4 \quad ; \quad x = -4 - 4$

$\Rightarrow \quad x = 4 - 4 \quad ; \quad x = -8$

$\Rightarrow \quad x = 0$

$\therefore$ Solution set $= \{0, -8\}$

iii. $x^2 + 6x - 9 = 0$

Solution: $x^2 + 6x = 9 \quad \ldots\ldots\ldots (1)$

To find the number to complete the square. It is $\left(\frac{6}{2}\right)^2 = 3^2 = 9.$

Add this number to both sides:

$\Rightarrow \quad x^2 + 6x + 9 = 9 + 9 \quad \Rightarrow \quad x^2 + 6x + 9 = 18$

Write the left side as a perfect square:

$\Rightarrow \quad (x+3)^2 = 18 \quad \Rightarrow \quad x+3 = \pm\sqrt{18}$

$\Rightarrow \quad x+3 = \pm\sqrt{9\times 2} \quad \Rightarrow \quad x+3 = \pm 3\sqrt{2}$

Finally, solve for x.

$\Rightarrow \quad x = -3 \pm 3\sqrt{2}$

$\therefore \quad$ **Solution set** $= \{-3 \pm 3\sqrt{2}\}$

iv. $3x^2 + 12x + 8 = 0$

Solution:

To solve the equation $3x^2 + 12x + 8 = 0$ by completing the square, follow these steps:

Divide the entire equation by 3 to simplify the coefficients:

$\Rightarrow \quad x^2 + 4x + \frac{8}{3} = 0 \quad \Rightarrow \quad x^2 + 4x = -\frac{8}{3}$

Find the number to complete the square. It is $\left(\frac{4}{2}\right)^2 = 2^2 = 4.$

Add this number to both sides:

$\Rightarrow \quad x^2 + 4x + 4 = -\frac{8}{3} + 4 \quad \Rightarrow \quad x^2 + 4x + 4 = -\frac{8}{3} + \frac{12}{3}$

$\Rightarrow \quad x^2 + 4x + 4 = \frac{4}{3} \quad \Rightarrow \quad (x+2)^2 = \frac{4}{3}$

Solve for x.

$\Rightarrow \quad x + 2 = \pm\sqrt{\frac{4}{3}} \quad \Rightarrow \quad x+2 = \pm\frac{2}{\sqrt{3}}$

$\Rightarrow \quad x = -2 \pm \frac{2}{\sqrt{3}} \quad \Rightarrow \quad x = \frac{-2\sqrt{3} \pm 2}{\sqrt{3}}$

$\therefore \quad$ **Solution set** $= \left\{\frac{-2\sqrt{3} \pm 2}{\sqrt{3}}\right\}$

v. $x^2 + x + 1 = 0$

Solution: $x^2 + x = -1 \quad \ldots\ldots\ldots (1)$

To find the number to complete the square. It is $\left(\frac{1}{2}\right)^2 = \frac{1}{4}.$

Add this number to both sides:

$\Rightarrow \quad x^2 + x + \frac{1}{4} = -1 + \frac{1}{4} \quad \Rightarrow \quad x^2 + x + \frac{1}{4} = -\frac{3}{4}$

Write the left side as a perfect square:

$\Rightarrow \quad \left(x + \frac{1}{2}\right)^2 = -\frac{3}{4} \quad \Rightarrow \quad x + \frac{1}{2} = \pm\sqrt{-\frac{3}{4}} \quad \Rightarrow \quad x + \frac{1}{2} = \pm\frac{\sqrt{-3}}{2}$

$\Rightarrow \quad x = -\frac{1}{2} \pm \frac{\sqrt{-3}}{2} \quad \Rightarrow \quad x = \frac{-1 \pm \sqrt{-3}}{2}$

$\therefore \quad$ **Solution set** $= \left\{\frac{-1 \pm \sqrt{-3}}{2}\right\}$

vi. $4x^2 - 8x - 5 = 0$

Solution: $4x^2 - 8x - 5 = 0$

We first divide through by 4 the coefficient of x^2 to make the coefficient of x^2 equal to 1.

$\Rightarrow \quad x^2 - 2x - \frac{5}{4} = 0 \quad \Rightarrow \quad x^2 - 2x = \frac{5}{4}$

Half the coefficient of x is $\frac{-2}{2} = -1$ and its square is $(-1)^2 = 1$.

$\Rightarrow \quad x^2 - 2x + 1 = \frac{5}{4} + 1 \quad \Rightarrow \quad x^2 - 2x + 1 = \frac{9}{4} \quad \Rightarrow \quad (x-1)^2 = \frac{9}{4}$

Take the square root of both sides of the equation.

$\Rightarrow \quad x - 1 = \pm\sqrt{\frac{9}{4}} \quad \Rightarrow \quad x - 1 = \pm\frac{3}{2} \quad \Rightarrow \quad x = 1 \pm \frac{3}{2}$

This gives us two solutions:

$\Rightarrow \quad x = 1 + \frac{3}{2} = \frac{2}{2} + \frac{3}{2} = \frac{5}{2}$ and $x = 1 - \frac{3}{2} = \frac{2}{2} - \frac{3}{2} = -\frac{1}{2}$

$\therefore$ **Solution set** $= \left\{\frac{5}{2}, -\frac{1}{2}\right\}$

Q4. Solve the following equations by quadratic formula:

i. $x^2 - 9 = 0$

Solution:

To solve the quadratic equation $x^2 - 9 = 0$ using the quadratic formula, we first identify the coefficients:

$\Rightarrow \quad a = 1, \; b = 0, \; c = -9$

The quadratic formula is given by:

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting the values of a, b, and c into the formula, we get:

$\Rightarrow \quad x = \frac{-0 \pm \sqrt{0^2 - 4(1)(-9)}}{2(1)} \quad \Rightarrow \quad x = \frac{\pm\sqrt{36}}{2} \quad \Rightarrow \quad x = \frac{\pm 6}{2}$

This gives us two solutions:

$\Rightarrow \quad x = \frac{6}{2} = 3 \quad ; \quad x = \frac{-6}{2} = -3$

$\therefore$ **Solution set** $= \{3, -3\}$

ii. $2x^2 + 5x + 1 = 0$

Solution:

To solve the quadratic equation $2x^2 + 5x + 1 = 0$ using the quadratic formula, the coefficients are:

$\Rightarrow \quad a = 2, \quad b = 5, \quad c = 1$

The quadratic formula is given by:

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting the values of a, b, and c into the formula, we get:

$$\Rightarrow \quad x = \frac{-5 \pm \sqrt{5^2 - 4(2)(1)}}{2(2)} \quad \Rightarrow \quad x = \frac{-5 \pm \sqrt{25 - 8}}{4} \quad \Rightarrow \quad x = \frac{-5 \pm \sqrt{17}}{4}$$

$\therefore$ **Solution set** $= \left\{\frac{-5 \pm \sqrt{17}}{4}\right\}$

iii. $x^2 - 23x - 24 = 0$

Solution:

To solve the quadratic equation $x^2 - 23x - 24 = 0$ using the quadratic formula, the coefficients are:

$$\Rightarrow \quad a = 1, \quad b = -23, \quad c = -24$$

The quadratic formula is given by: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substituting the values of a, b, and c into the formula, we get:

$$\Rightarrow \quad x = \frac{-(-23) \pm \sqrt{(-23)^2 - 4(1)(-24)}}{2(1)}$$

$$\Rightarrow \quad x = \frac{23 \pm \sqrt{529 + 96}}{2} \quad \Rightarrow \quad x = \frac{23 \pm \sqrt{625}}{2} \quad \Rightarrow \quad x = \frac{23 \pm 25}{2}$$

Now we have two solutions:

$$\Rightarrow \quad x = \frac{23 + 25}{2} = \frac{48}{2} = 24 \quad ; \quad x = \frac{23 - 25}{2} = \frac{-2}{2} = -1$$

$\therefore$ **Solution set** $= \{-1, 24\}$

iv. $(x + 1)^2 = (2x - 1)^2$

Solution:

$$\Rightarrow \quad x^2 + 2x + 1 = 4x^2 - 4x + 1$$

Subtracting $x^2 + 2x + 1$ from both sides:

$$\Rightarrow \quad 0 = 4x^2 - 4x + 1 - (x^2 + 2x + 1)$$

$$\Rightarrow \quad 0 = 4x^2 - 4x + 1 - x^2 - 2x - 1 \quad \Rightarrow \quad 0 = 3x^2 - 6x$$

Divide by 3.

$$\Rightarrow \quad 0 = x^2 - 2x \quad \Rightarrow \quad x^2 - 2x = 0$$

Applying the quadratic formula where $a = 1$, $b = -2$, and $c = 0$:

$$\Rightarrow \quad x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(0)}}{2(1)} \quad \Rightarrow \quad x = \frac{2 \pm \sqrt{4}}{2} \quad \Rightarrow \quad x = \frac{2 \pm 2}{2}$$

This gives us two solutions:

$$\Rightarrow \quad x = \frac{2 + 2}{2} = \frac{4}{2} = 2 \quad ; \quad x = \frac{2 - 2}{2} = \frac{0}{2} = 0$$

$\therefore$ **Solution set** $= \{0, 2\}$

v. $\frac{x+1}{2}-\frac{x(x+2)}{3}=0$

Solution:

Multiply both sides by 3, we get: $\frac{3(x+1)}{6}-\frac{2x(x+2)}{6}=0$

Simplify and combine like terms:

$\Rightarrow \quad \frac{3x+3-2x^2-4x}{6}=0$; (Multiply both sides by 6)

$\Rightarrow \quad 3x+3-2x^2-4x=0$

$\Rightarrow \quad -2x^2-x+3=0$

Now we can apply the quadratic formula with $a=-2,\ b=-1$, and $c=3$.

$\Rightarrow \quad x=\frac{-(-1)\pm\sqrt{(-1)^2-4(-2)(3)}}{2(-2)}$

$\Rightarrow \quad x=\frac{1\pm\sqrt{1+24}}{-4} \quad \Rightarrow \quad x=\frac{1\pm\sqrt{25}}{-4} \quad \Rightarrow \quad x=\frac{1\pm 5}{-4}$

This gives us two solutions:

$\Rightarrow \quad x=\frac{1+5}{-4}=\frac{6}{-4}=-\frac{3}{2}$; $x=\frac{1-5}{-4}=\frac{-4}{-4}=1$

$\therefore$ **Solution set** $=\left\{1,-\frac{3}{2}\right\}$

vi. $(x-2)(x-6)=(2x+1)(x+1)$

Solution:

To solve the equation $(x-2)(x-6)=(2x+1)(x+1)$, first expand both sides:

$\Rightarrow \quad x^2-6x-2x+12=2x^2+x+2x+1$

Simplify and combine like terms:

$\Rightarrow \quad x^2-8x+12=2x^2+3x+1$

Subtract $x^2-8x+12$ from both sides to set the equation to zero:

$\Rightarrow \quad 0=2x^2+3x+1-(x^2-8x+12)$

$\Rightarrow \quad 0=2x^2+3x+1-x^2+8x-12$

$\Rightarrow \quad 0=x^2+11x-11$

$\Rightarrow \quad x^2+11x-11=0$

Now apply the quadratic formula with $a=1,\ b=11$, and $c=-11$.

$\Rightarrow \quad x=\frac{-11\pm\sqrt{11^2-4(1)(-11)}}{2(1)}$

$\Rightarrow \quad x=\frac{-11\pm\sqrt{121+44}}{2} \quad \Rightarrow \quad x=\frac{-11\pm\sqrt{165}}{2}$

$\therefore$ **Solution set** $=\left\{\frac{-11\pm\sqrt{165}}{2}\right\}$

Q5. Solve $x^2 + 6x = -9$ by graphing and factoring method.

Solution: $x^2 + 6x = -9$

- **Factoring method:**

$\Rightarrow$ $x^2 + 6x + 9 = 0$

$\Rightarrow$ $x^2 + 3x + 3x + 9 = 0$ $\Rightarrow$ $x(x+3) + 3(x+3) = 0$

$\Rightarrow$ $(x+3)(x+3) = 0$ $\Rightarrow$ $(x+3)^2 = 0$

Set the factor equal to zero and solve for x:

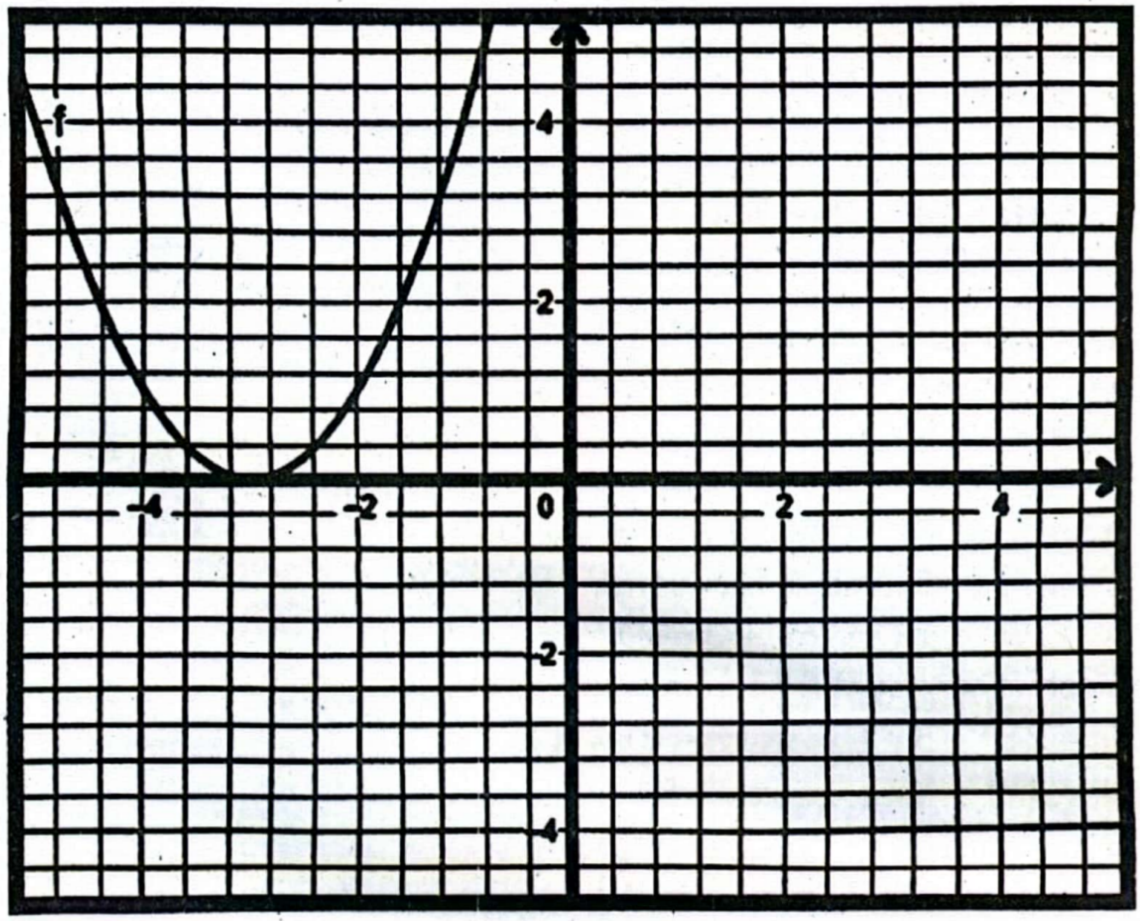
$\Rightarrow$ $x + 3 = 0$ and $x + 3 = 0$

$\Rightarrow$ $x = -3$ and $x = -3$

$\therefore$ Solution set $= \{-3, -3\}$

- **Graphing method:**

Now, let's solve the equation by graphing. We can rewrite the equation as $x^2 + 6x + 9 = 0$. Let $y = x^2 + 6x + 9$. We want to find the x-value(s) where $y = 0$.



Q6. Graph the function $y = x^2 + 2x + 4$ and verifying solution by completing square method.

Solution:

Let's graph the function $y = x^2 + 2x + 4$ and verify the solution by completing the square method.

First, let's complete the square for the quadratic expression $x^2 + 2x + 4$.

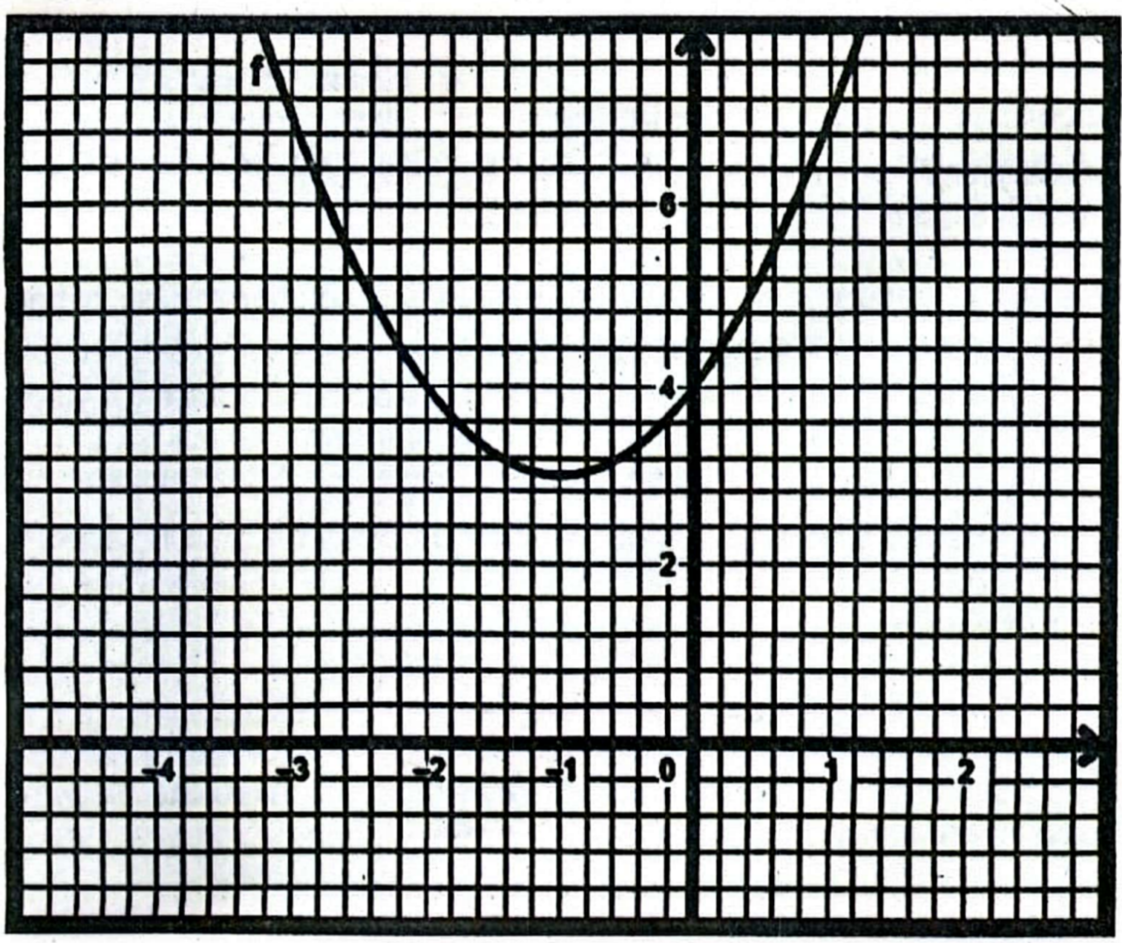
$\Rightarrow$ $y = x^2 + 2x + 4$

$\Rightarrow$ $y = (x^2 + 2x + 1) + 4 - 1$

$\Rightarrow \quad y = (x+1)^2 + 3$

From the completed square form, we can see that the vertex of the parabola is at $(-1, 3)$. Since the coefficient of the x^2 term is positive, the parabola opens upwards.

- **Graph:**



Q7. Explain each term:

i.	**Solution**	ii.	**root**
iii.	**Zero of a function**	iv.	**x-intercept**

Solution:

i. **Solution:** A solution is a value that, when substituted into an equation, makes the equation true. For example, in the equation $x + 2 = 5$, the solution is $x = 3$ because $3 + 2 = 5$.

ii. **Root:** A root is a solution to an equation, typically a polynomial equation. In the context of a function $f(x)$, a root is a value x such that $f(x) = 0$. For example, the roots of the equation $x^2 - 5x + 6 = 0$ are $x = 2$ and $x = 3$.

iii. **Zero of a function:** A zero of a function $f(x)$ is a value x for which $f(x) = 0$. In other words, it's the x-value where the function's graph intersects or touches the x-axis. Zeros of a function are also called roots of the function.

iv. **x-intercept:** An x-intercept is the point where the graph of a function intersects the x-axis. At this point, the y-coordinate is zero. So, an x-intercept is represented as $(x, 0)$, where x is the value where the graph