Exercise 1.3

➢ **Solve:**

Q1. $x^2 + 7 = 0$

Solution: $x^2 + 7 = 0$

$\Rightarrow$ $x^2 = -7$

Now we take the square root of both sides:

$\Rightarrow$ $x = \pm\sqrt{-7} = \pm\sqrt{-1 \times 7} = \pm\sqrt{-1} \times \sqrt{7}$; $[\because \ i = \sqrt{-1}]$

$\Rightarrow$ $x = \pm\sqrt{7}i$

Solution set $= \{\pm\sqrt{7}i\}$

Q2. $x^2 + 9 = 0$

Solution: $x^2 + 9 = 0$

$\Rightarrow$ $x^2 = -9$

Now we take the square root of both sides:

$\Rightarrow$ $x = \pm\sqrt{-9} = \pm\sqrt{-1 \times 9} = \pm 3\sqrt{-1}$; $[\because \ i = \sqrt{-1}]$

$\Rightarrow$ $x = \pm 3i$

Solution set $= \{\pm 3i\}$

Q3. $x^2 + 100 = 0$

Solution: $x^2 + 100 = 0$

$\Rightarrow$ $x^2 = -100$

Now we take the square root of both sides:

$\Rightarrow$ $x = \pm\sqrt{-100} = \pm\sqrt{-1 \times 100} = \pm 10\sqrt{-1}$; $[\because \ i = \sqrt{-1}]$

$\Rightarrow$ $x = \pm 10i$

Solution set $= \{\pm 10i\}$

$\Rightarrow$ $x = \pm\sqrt{-100}$

➢ **Determine whether the given complex number is a solution of the equation.**

Q4. $1 + 2i, \ x^2 - 2x + 5 = 0$

Solution: $1 + 2i, \ x^2 - 2x + 5 = 0$

To determine whether the given complex number $1 + 2i$ is a solution of the equation $x^2 - 2x + 5 = 0$, we substitute $x = 1 + 2i$ into the equation and check if it satisfies the equation.

$\Rightarrow$ $x^2 - 2x + 5 = (1 + 2i)^2 - 2(1 + 2i) + 5$ ……… (i)

First, we expand $(1 + 2i)^2$.

$\Rightarrow \quad (1+2i)^2 = (1+2i)(1+2i) = 1+2i+2i+4i^2$
$= 1+4i-4 = -3+4i \quad ; \quad [\because \ i^2 = -1]$

Now, we substitute this back into the equation (i).

$\Rightarrow \quad (-3+4i) - 2(1+2i) + 5 = -3+4i-2-4i+5$
$= -5+5+4i-4i = 0$

Since the result is 0, the complex number $1+2i$ is a solution of the equation.

$\Rightarrow \quad x^2 - 2x + 5 = 0$

The final answer is **Yes.**

Q5. $1-2i, \ x^2 - 2x + 5 = 0$

Solution:

To determine whether the given complex number $1-2i$ is a solution of the equation $x^2 - 2x + 5 = 0$, we substitute $x = 1-2i$ into the equation and check if it satisfies the equation.

$\Rightarrow \quad x^2 - 2x + 5 = (1-2i)^2 - 2(1-2i) + 5 \quad \ldots\ldots\ldots \text{(i)}$

First, we expand $(1-2i)^2$.

$\Rightarrow \quad (1-2i)^2 = (1-2i)(1-2i) = 1-2i-2i+4i^2$
$= 1-4i-4 = -3-4i \quad ; \quad [\because i^2 = -1]$

Now, we substitute this back into the equation (i).

$\Rightarrow \quad (-3-4i) - 2(1-2i) + 5 = -3-4i-2+4i+5$
$= -5+5-4i+4i = 0$

Since the result is 0, the complex number $1-2i$ is a solution of the equation.

$\Rightarrow \quad x^2 - 2x + 5 = 0$

The final answer is **Yes.**

Q6. $1-i, \ x^2 + 2x + 2 = 0$

Solution:

To determine whether the given complex number $1-i$ is a solution of the equation $x^2 + 2x + 2 = 0$, we substitute $x = 1-i$ into the equation and check if it satisfies the equation.

$\Rightarrow \quad x^2 + 2x + 2 = (1-i)^2 + 2(1-i) + 2 \quad \ldots\ldots\ldots \text{(i)}$

First, we expand $(1-i)^2$.

$\Rightarrow \quad (1-i)^2 = (1-i)(1-i) = 1-i-i+i^2$
$= 1-2i-1 = -2i \quad ; \quad [\because \ i^2 = -1]$

Now, we substitute this back into the equation (i).

$\Rightarrow \quad (-2i) + 2(1-i) + 2 = -2i+2-2i+2 = 4-4i \neq 0$

Since the result is not 0, the complex number $1-i$ is not a solution of the equation $x^2 + 2x + 2 = 0$.

The final answer is **No.**

Q7. $i\ ,\ x^2 + 1 = 0$

Solution:

To determine whether the given complex number i is a solution of the equation $x^2 + 1 = 0$, we substitute $x = i$ into the equation and check if it satisfies the equation.

$\Rightarrow \quad x^2 + 1 = (i)^2 + 1$

Since $i^2 = -1$, we have:

$\Rightarrow \quad (i)^2 + 1 = -1 + 1 = 0$

Since the result is 0, the complex number i is a solution of the equation

$\Rightarrow \quad x^2 + 1 = 0$

The final answer is **Yes.**

➢ Factorize the expressions:

Q8. $x^2 + 16$

Solution: $x^2 + 16$

We know that $i^2 = -1$, so we can write 16 as $-16i^2$.

Then,

$\Rightarrow \quad x^2 + 16 = x^2 - (-16) = x^2 - 16i^2 = x^2 - (4i)^2 = (x - 4i)(x + 4i)$

Q9. $a^2 + b^2$

Solution: $a^2 + b^2$

We know that $i^2 = -1$, so we can write b^2 as $-b^2 i^2$.

Then,

$\Rightarrow \quad a^2 + b^2 = a^2 - (-b^2) = a^2 - b^2 i^2 = a^2 - (bi)^2 = (a - bi)(a + bi)$

Q10. $x^2 + 25y^2$

Solution:

We know that $i^2 = -1$, so we can write $25y^2$ as $-25y^2 i^2$.

Then, $x^2 + 25y^2 = x^2 - (-25y^2) = x^2 - 25y^2 i^2 = x^2 - (5yi)^2$

$= (x - 5yi)(x + 5yi)$

➢ Solve the following system of linear equations.

Q11. $z - 4w = 3i \ ; \quad 2z + 3w = 11 - 5i$

Solution: $z - 4w = 3i \ ; \quad 2z + 3w = 11 - 5i$

We are given the system of linear equations:

$$\Rightarrow \begin{cases} z - 4w = 3i & \dots\dots\dots \text{(i)} \\ 2z + 3w = 11 - 5i & \dots\dots\dots \text{(ii)} \end{cases}$$

Multiply the first equation by -2 on both sides:

$\Rightarrow \quad -2(z - 4w) = -2(3i)$

$\Rightarrow \quad -2z + 8w = -6i$

Now add this to the second equation:

$\Rightarrow \quad (-2z + 8w) + (2z + 3w) = -6i + (11 - 5i)$

$\Rightarrow \quad 11w = 11 - 11i$

Divide by 11.

$\Rightarrow \quad w = 1 - i$

Now substitute $w = 1 - i$ into the first equation:

$\Rightarrow \quad z - 4(1 - i) = 3i \quad \Rightarrow \quad z - 4 + 4i = 3i$

$\Rightarrow \quad z = 4 - 4i + 3i \quad \Rightarrow \quad z = 4 - i$

So, the solution is $z = 4 - i$ and $w = 1 - i$.

Q12. $3z + (2 + i)w = 11 - i \;;\; (2 - i)z - w = -1 + i$

Solution: $3z + (2 + i)w = 11 - i \;;\; (2 - i)z - w = -1 + i$

We are given the system of linear equations:

$$\Rightarrow \begin{cases} 3z + (2 + i)w = 11 - i & \dots\dots\dots (i) \\ (2 - i)z - w = -1 + i & \dots\dots\dots (ii) \end{cases}$$

Multiply the second equation by $2 + i$ on both sides:

$\Rightarrow \quad (2 + i)(2 - i)z - (2 + i)w = (2 + i)(-1 + i)$

$\Rightarrow \quad (4 - i^2)z - (2 + i)w = -2 + 2i - i + i^2$

$\Rightarrow \quad (4 + 1)z - (2 + i)w = -2 + 2i - i - 1 \quad ; \quad [\because \; i = \sqrt{-1}]$

$\Rightarrow \quad 5z - (2 + i)w = -3 + i \quad \dots\dots\dots (iii)$

Now we have from (i) and (iii).

$$\Rightarrow \begin{cases} 3z + (2 + i)w = 11 - i \\ 5z - (2 + i)w = -3 + i \end{cases}$$

Add (i) and (iii).

$\Rightarrow \quad [3z + (2 + i)w] + [5z - (2 + i)w] = (11 - i) + (-3 + i)$

$\Rightarrow \quad 8z = 8 \quad \Rightarrow \quad z = 1$

Now substitute $z = 1$ into the second equation.

$\Rightarrow \quad (2 - i)(1) - w = -1 + i \quad \Rightarrow \quad 2 - i - w = -1 + i$

$\Rightarrow \quad -w = -1 + i - 2 + i \quad \Rightarrow \quad -w = -3 + 2i \quad \Rightarrow \quad w = 3 - 2i$

So the solution is $z = 1$ and $w = 3 - 2i$.

Q13. In an electrical circuit, the flow of the electric current I, the impedance Z and the voltage E, are related by the formula $E = IZ$.

a. Find I, given the values:

i. $E = (70 + 220\,i)$ **volts,** $Z = (16 + 8\,i)$ **ohms**

ii. $E = (85 + 110\,i)$ **volts,** $Z = (3 - 4\,i)$ **ohms**

b. Find Z given the value:

i. $E = (-50 + 100\,i)$ **volts,** $I = (-6 - 2\,i)$**amp**

ii. $E = (100 + 10\,i)$ **volts,** $I = (-8 + 3\,i)$**amp**

c. Evaluate $\dfrac{1}{z - z^2}$ **when** $z = \dfrac{1 - i}{10}$.

Solution:

a. **Find I, given the values:**

i. $E = (70 + 220\,i)$ volts, $Z = (16 + 8\,i)$ ohms

Solution: $I = \frac{E}{Z} = \frac{70 + 220j}{16 + 8j}$

Multiply the numerator & denominator by the conjugate of the denominator:

$$\Rightarrow \quad I = \frac{(70 + 220i)(16 - 8i)}{(16 + 8i)(16 - 8i)} = \frac{1120 - 560j + 3520i - 1760i^2}{256 - 64i^2}$$

Since, $i^2 = -1$.

$$= \frac{1120 - 560j + 3520i - 1760(-1)}{256 - 64i^2}$$

$$\Rightarrow \quad I = \frac{1120 + 1760 + (3520 - 560)i}{256 + 64} = \frac{2880 + 2960i}{320}$$

$$\Rightarrow \quad I = \frac{2880}{320} + \frac{2960}{320}i = 9 + \frac{37}{4}i$$

a. **Find I, given the values:**

ii. $E = (85 + 110\,i)$ volts, $Z = (3 - 4\,i)$ ohms

Solution: $I = \frac{E}{Z} = \frac{85 + 110i}{3 - 4i}$

Multiply the numerator & denominator by the conjugate of the denominator:

$$\Rightarrow \quad I = \frac{(85 + 110i)(3 + 4i)}{(3 - 4i)(3 + 4i)} = \frac{255 + 340i + 330i + 440i^2}{9 - 16i^2}$$

Since, $i^2 = -1$.

$$= \frac{255 + 340i + 330i + 440(-1)}{9 - 16(-1)}$$

$$\Rightarrow \quad I = \frac{255 - 440 + (340 + 330)i}{9 + 16} = \frac{-185 + 670i}{25} = -7.4 + 26.8i$$

$$\Rightarrow \quad I = \frac{-185}{258} + \frac{670}{25}i = \frac{-37}{5} + \frac{134}{5}i$$

b. **Find Z given the value:**

i. $E = (-50 + 100\,i)$ volts, $I = (-6 - 2\,i)$ amp

Solution: $Z = \frac{E}{I} = \frac{-50 + 100i}{-6 - 2i}$

Multiply the numerator & denominator by the conjugate of the denominator:

$$\Rightarrow \quad Z = \frac{(-50 + 100i)(-6 + 2i)}{(-6 - 2i)(-6 + 2i)} = \frac{300 - 100i - 600i + 200i^2}{36 - 4i^2}$$

Since, $i^2 = -1$.

$$\Rightarrow \quad Z = \frac{300 - 100i - 600i + 200(-1)}{36 - 4(-1)} = \frac{300 - 200 - 700i}{36 + 4} = \frac{100 - 700i}{40}$$

$$\Rightarrow \quad Z = \frac{100}{40} - \frac{700}{40}i = \frac{5}{2} - \frac{35}{2}i$$

b. Find Z given the value:

ii. $E = (100 + 10\,i)$ volts, $I = (-8 + 3\,i)$ amp

Solution:

$$\Rightarrow \quad Z = \frac{E}{I} = \frac{100 + 10i}{-8 + 3i}$$

Multiply the numerator & denominator by the conjugate of the denominator:

$$\Rightarrow \quad Z = \frac{(100 + 10i)(-8 - 3i)}{(-8 + 3i)(-8 - 3i)} = \frac{-800 - 300i - 80i - 30i^2}{64 - 9i^2}$$

Since, $i^2 = -1$.

$$\Rightarrow \quad Z = \frac{-800 - 300i - 80i - 30(-1)}{64 - 9(-1)} = \frac{-800 + 30 - 380i}{64 + 9}$$

$$\Rightarrow \quad Z = \frac{-770 - 380i}{73} = -\frac{770}{73} - \frac{380}{73}i$$

c. Evaluate $\frac{1}{z - z^2}$ when $z = \frac{1 - i}{10}$.

Solution:

$$\Rightarrow \quad z = \frac{1 - i}{10}$$

$$\Rightarrow \quad z^2 = \left(\frac{1 - i}{10}\right)^2 = \frac{(1 - i)^2}{100} = \frac{1 - 2i + i^2}{100} = \frac{1 - 2i - 1}{100} = \frac{-2i}{100} = -\frac{i}{50}$$

$$\Rightarrow \quad z - z^2 = \frac{1 - i}{10} - \left(-\frac{i}{50}\right) = \frac{1 - i}{10} + \frac{i}{50} = \frac{5(1 - i) + i}{50} = \frac{5 - 5i + i}{50} = \frac{5 - 4i}{50}$$

Now, evaluate $\frac{1}{z - z^2}$.

$$\Rightarrow \quad \frac{1}{z - z^2} = \frac{1}{\frac{5 - 4i}{50}} = \frac{50}{5 - 4i}$$

Multiply the numerator & denominator by the conjugate of the denominator:

$$\Rightarrow \quad \frac{1}{z - z^2} = \frac{50(5 + 4i)}{(5 - 4i)(5 + 4i)} = \frac{250 + 200i}{25 - 16i^2} = \frac{250 + 200i}{25 - 16(-1)} \quad ; \quad [\because \; i^2 = -1]$$

$$\Rightarrow \quad \frac{1}{z - z^2} = \frac{250 + 200i}{25 + 16} = \frac{250 + 200i}{41} = \frac{250}{41} + \frac{200}{41}i$$

So, $\frac{1}{z - z^2} = \frac{250}{41} + \frac{200}{41}i$

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