Exercise 1.2

Q1. Find the additive inverse of each complex number.

a. $-4+5i$

Solution: $-4+5i$

Let $a+bi$ the additive inverse of $-4+5i$ then by additive inverse rule:

$(-4+5i)+(a+bi)=0+0i$

$(-4+a)+(5i+bi)=0+0i$

$\Rightarrow$ $(-4+a)+(5+b)i=0+0i$

$\Rightarrow$ $-4+a=0$ and $5+b=0$

$\Rightarrow$ $a=4$ and $b=-5$

$\therefore$ $a+bi=4-5i$

Hence, the additive inverse of $-4+5i$ is $4-5i$.

b. $-3-3i$

Solution: $-3-3i$

Let $a+bi$ the additive inverse of $-3-3i$ then by additive inverse rule:

$(-3-3i)+(a+bi)=0+0i$

$(-3+a)+(-3i+bi)=0+0i$

$\Rightarrow$ $(-3+a)+(-3+b)i=0+0i$

$\Rightarrow$ $-3+a=0$ and $-3+b=0$

$\Rightarrow$ $a=3$ and $b=3$

$\therefore$ $a+bi=3+3i$

Hence, the additive inverse of $-3-3i$ is $3+3i$.

c. $5-5i$

Solution: $5-5i$

Let $a+bi$ the additive inverse of $5-5i$ then by additive inverse rule:

$(5-5i)+(a+bi)=0+0i$

$(5+a)+(-5i+bi)=0+0i$

$\Rightarrow$ $(5+a)+(-5+b)i=0+0i$

$\Rightarrow$ $5+a=0$ and $-5+b=0$

$\Rightarrow$ $a=-5$ and $b=5$

$\therefore$ $a+bi=-5+5i$

Hence, the additive inverse of $5-5i$ is $-5+5i$.

d. $4i$

Solution:

Let $a+bi$ the additive inverse of $4i$ then by additive inverse rule:

$(0+4i)+(a+bi)=0+0i$

$(0+a)+(4i+bi)=0+0i$

$\Rightarrow \quad (a) + (4 + b)i = 0 + 0i$

$\Rightarrow \quad a = 0$ and $4 + b = 0$

$\Rightarrow \quad a = 0$ and $b = -4$

$\therefore \quad a + bi = 0 - 4i = -4i$

Hence, the additive inverse of $4i$ is $-4i$.

Q2. <u>Show that each pair of complex numbers are multiplicative inverse of each other.</u>

a. $2 + 3i, \dfrac{2 - 3i}{13}$

Solution: $2 + 3i, \dfrac{2 - 3i}{13}$

To show that the pair of complex numbers $2 + 3i$ and $\dfrac{2-3i}{13}$ are multiplicative inverses of each other, we need to multiply them together and see if the result is equal to 1.

Let $z_1 = 2 + 3i$ and $z_2 = \dfrac{2-3i}{13}$. Then,

$\Rightarrow \quad z_1 \cdot z_2 = (2 + 3i) \times \dfrac{2 - 3i}{13} = \dfrac{(2 + 3i)(2 - 3i)}{13}$ ……… (i)

We multiply the complex numbers in the numerator:

$\Rightarrow \quad (2 + 3i)(2 - 3i) = 2(2) + 2(-3i) + 3i(2) + 3i(-3i) = 4 - 6i + 6i - 9i^2$

Since $i^2 = -1$, we have:

$\Rightarrow \quad 4 - 6i + 6i - 9(-1) = 4 + 9 = 13$

So the expression (i) becomes: $\quad z_1 \cdot z_2 = \dfrac{13}{13} = 1$

Since the product of the two complex numbers is 1, they are multiplicative inverses of each other.

b. $5 - 4i, \dfrac{5 + 4i}{41}$

Solution: $5 - 4i, \dfrac{5 + 4i}{41}$

To show that the pair of complex numbers $5 - 4i$ and $\dfrac{5+4i}{41}$ are multiplicative inverses of each other, we need to multiply them together and see if the result is equal to 1.

Let $z_1 = 5 - 4i$ and $z_2 = \dfrac{5+4i}{41}$. Then,

$\Rightarrow \quad z_1 \cdot z_2 = (5 - 4i) \cdot \dfrac{5 + 4i}{41} = \dfrac{(5 - 4i)(5 + 4i)}{41}$ ……… (i)

We multiply the complex numbers in the numerator:

$\Rightarrow \quad (5 - 4i)(5 + 4i) = 5(5) + 5(4i) - 4i(5) - 4i(4i)$

$= 25 + 20i - 20i - 16i^2$

Since $i^2 = -1$, we have:

$\Rightarrow \quad 25 + 20i - 20i - 16(-1) = 25 + 16 = 41$

So the expression (i) becomes: $\quad z_1 \cdot z_2 = \frac{41}{41} = 1$

Since the product of the two complex numbers is 1, they are multiplicative inverses of each other.

c. $\quad 6 + 8i, \frac{3 - 4i}{50}$

Solution: $\quad 6 + 8i, \frac{3 - 4i}{50}$

To show that the pair of complex numbers $6 + 8i$ and $\frac{3 - 4i}{50}$ are multiplicative inverses of each other, we need to multiply them together and see if the result is equal to 1.

Let $z_1 = 6 + 8i$ and $z_2 = \frac{3 - 4i}{50}$. Then,

$$\Rightarrow \quad z_1 \cdot z_2 = (6 + 8i) \cdot \frac{3 - 4i}{50} = \frac{(6 + 8i)(3 - 4i)}{50} \qquad \text{......... (i)}$$

We multiply the complex numbers in the numerator:

$$\Rightarrow \quad (6 + 8i)(3 - 4i) = 6(3) + 6(-4i) + 8i(3) + 8i(-4i)$$
$$= 18 - 24i + 24i - 32i^2$$

Since $i^2 = -1$, we have:

$\Rightarrow \quad 18 - 24i + 24i - 32(-1) = 18 + 32 = 50$

So the expression (i) becomes: $\quad z_1 \cdot z_2 = \frac{50}{50} = 1$

Since the product of the two complex numbers is 1, they are multiplicative inverses of each other.

Q3. <u>Find the multiplicative inverse of each complex number.</u>

a. $\quad 1 + i$

Solution: $\quad 1 + i$

The multiplicative inverse of a complex number z is $\frac{1}{z}$.

The reciprocal of $1 + i$ is $\frac{1}{1+i}$. The complex conjugate of $1 + i$ is $1 - i$.

$$\Rightarrow \quad \frac{1}{1+i} = \frac{1}{1+i} \times \frac{1-i}{1-i} = \frac{1-i}{(1+i)(1-i)}$$

Since $i^2 = -1$, we have:

$$= \frac{1-i}{(1)^2 - i^2} = \frac{1-i}{1-(-1)} = \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i$$

Thus, the multiplicative inverse of $1 + i$ is $\frac{1}{2} - \frac{1}{2}i$.

➤ **Verification:**

Now, we check the answer by multiplying it by the original number:

$$\Rightarrow \quad (1+i)\left(\frac{1}{2}-\frac{1}{2}i\right) = 1\left(\frac{1}{2}-\frac{1}{2}i\right) + i\left(\frac{1}{2}-\frac{1}{2}i\right)$$

$$= \frac{1}{2} - \frac{1}{2}i + \frac{1}{2}i - \frac{1}{2}i^2 = \frac{1}{2} - \frac{1}{2}i + \frac{1}{2}i + \frac{1}{2}$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

Since the product is **1**, the multiplicative inverse is correct.

b. $7 - 3i$

Solution: $7 - 3i$

The multiplicative inverse of a complex number z is $\frac{1}{z}$. The reciprocal of $7 - 3i$ is $\frac{1}{7-3i}$. The complex conjugate of $7 - 3i$ is $7 + 3i$.

$$\Rightarrow \quad \frac{1}{7-3i} = \frac{1}{7-3i} \times \frac{7+3i}{7+3i} = \frac{7+3i}{(7-3i)(7+3i)}$$

Since $i^2 = -1$, we have:

$$= \frac{7+3i}{(7)^2-(3i)^2} = \frac{7+3i}{49-9i^2} = \frac{7+3i}{49-9(-1)}$$

$$= \frac{7+3i}{49+9} = \frac{7+3i}{58} = \frac{7}{58} + \frac{3}{58}i$$

Thus, the multiplicative inverse of $7 - 3i$ is $\frac{7}{58} + \frac{3}{58}i$.

➤ **Verification:**

Now, we check the answer by multiplying it by the original number:

$$\Rightarrow \quad (7-3i)\left(\frac{7}{58}+\frac{3}{58}i\right) = 7\left(\frac{7}{58}+\frac{3}{58}i\right) - 3i\left(\frac{7}{58}+\frac{3}{58}i\right)$$

$$= \frac{49}{58} + \frac{21}{58}i - \frac{21}{58}i - \frac{9}{58}i^2$$

$$= \frac{49}{58} + \frac{21}{58}i - \frac{21}{58}i + \frac{9}{58}$$

$$= \frac{49}{58} + \frac{9}{58} = \frac{58}{58} = 1$$

Since the product is **1**, the multiplicative inverse is correct.

c. $10 - 12i$

Solution: $10 - 12i$

The multiplicative inverse of a complex number z is $\frac{1}{z}$. The reciprocal of $10 - 12i$ is $\frac{1}{10-12i}$. The complex conjugate of $10 - 12i$ is $10 + 12i$.

$$\Rightarrow \quad \frac{1}{10-12i} = \frac{1}{10-12i} \times \frac{10+12i}{10+12i} = \frac{10+12i}{(10-12i)(10+12i)}$$

Since $i^2 = -1$, we have:

$$= \frac{10+12i}{(10)^2-(12i)^2} = \frac{10+12i}{100-144i^2} = \frac{10+12i}{100-144(-1)} = \frac{10+12i}{100+144}$$

$$= \frac{10+12i}{244} = \frac{10}{244} + \frac{12}{244}i = \frac{5}{122} + \frac{3}{61}i$$

Thus, the multiplicative inverse of $10-12i$ is $\frac{5}{122} + \frac{3}{61}i$.

➤ **Verification:**

Now, we check the answer by multiplying it by the original number:

$$\Rightarrow \quad (10-12i)\left(\frac{5}{122} + \frac{3}{61}i\right) = 10\left(\frac{5}{122} + \frac{6}{122}i\right) - 12i\left(\frac{5}{122} + \frac{6}{122}i\right)$$

$$= \frac{50}{122} + \frac{60}{122}i - \frac{60}{122}i - \frac{72}{122}i^2 = \frac{50}{122} + \frac{60}{122}i - \frac{60}{122}i + \frac{72}{122}$$

$$= \frac{50}{122} + \frac{72}{122} = \frac{122}{122} = 1$$

Since the product is **1**, the multiplicative inverse is correct.

d. $\frac{2}{5-i}$

Solution: $\frac{2}{5-i}$

The multiplicative inverse of a complex number z is $\frac{1}{z}$.

The reciprocal of $\frac{2}{5-i}$ is $\frac{1}{\frac{2}{5-i}} = \frac{5-i}{2}$.

Now, we need to write this in the standard form $a+bi$.

$$\Rightarrow \quad \frac{5-i}{2} = \frac{5}{2} - \frac{1}{2}i$$

Thus, the multiplicative inverse of $\frac{2}{5-i}$ is $\frac{5}{2} - \frac{1}{2}i$.

➤ **Verification:**

Now, we check the answer by multiplying it by the original number:

$$\Rightarrow \quad \left(\frac{2}{5-i}\right)\left(\frac{5-i}{2}\right) = \frac{2(5-i)}{2(5-i)} = 1$$

Since the product is 1, the multiplicative inverse is correct.

The final answer is $\frac{5}{2} - \frac{1}{2}i$.

e. $\frac{-i}{2-3i}$

Solution: $\frac{-i}{2-3i}$

The multiplicative inverse of a complex number z is $\frac{1}{z}$.

The reciprocal of $\frac{-i}{2-3i}$ is $\frac{1}{\frac{-i}{2-3i}} = \frac{2-3i}{-i}$.

To simplify this expression, we need to multiply both the numerator and the denominator by the complex conjugate of the denominator. The complex conjugate of $-i$ is i.

$$\Rightarrow \quad \frac{2-3i}{-i} = \frac{2-3i}{-i} \times \frac{i}{i} = \frac{(2-3i)i}{-i^2} = \frac{2i-3i^2}{-(-1)} = \frac{2i-3(-1)}{1} = 2i+3 = 3+2i$$

Thus, the multiplicative inverse of $\frac{-i}{2-3i}$ is $3+2i$.

➢ **Verification:**

Now, we check the answer by multiplying it by the original number:

$$\Rightarrow \quad \left(\frac{-i}{2-3i}\right)(3+2i) = \frac{-i(3+2i)}{2-3i} = \frac{-3i-2i^2}{2-3i} = \frac{-3i-2(-1)}{2-3i} = \frac{2-3i}{2-3i} = 1$$

Since the product is 1, the multiplicative inverse is correct.

f. $a-bi$

Solution: $a-bi$

The multiplicative inverse of a complex number z is $\frac{1}{z}$.

The reciprocal of $a-bi$ is $\frac{1}{a-bi}$.

The complex conjugate of $a-bi$ is $a+bi$.

$$\Rightarrow \quad \frac{1}{a-bi} = \frac{1}{a-bi} \times \frac{a+bi}{a+bi} = \frac{a+bi}{(a-bi)(a+bi)}$$

Using the difference of squares formula, $(x-y)(x+y) = x^2 - y^2$, we have:

$$\Rightarrow \quad (a-bi)(a+bi) = a^2 - (bi)^2 = a^2 - b^2i^2$$

Since $i^2 = -1$, we have:

$$\Rightarrow \quad a^2 - b^2i^2 = a^2 - b^2(-1) = a^2 + b^2$$

So,

$$\Rightarrow \quad \frac{a+bi}{(a)^2-(bi)^2} = \frac{a+bi}{a^2-b^2i^2} = \frac{a+bi}{a^2-b^2(-1)} = \frac{a+bi}{a^2+b^2} = \frac{a}{a^2+b^2} + \frac{b}{a^2+b^2}i$$

Thus, the multiplicative inverse of $a-bi$ is $\frac{a}{a^2+b^2} + \frac{b}{a^2+b^2}i$.

➢ **Verification:**

Now, we check the answer by multiplying it by the original number:

$$\Rightarrow \quad (a-bi)\left(\frac{a}{a^2+b^2}+\frac{b}{a^2+b^2}i\right) = a\left(\frac{a}{a^2+b^2}+\frac{b}{a^2+b^2}i\right) - bi\left(\frac{a}{a^2+b^2}+\frac{b}{a^2+b^2}i\right)$$

$$= \frac{a^2}{a^2+b^2}+\frac{ab}{a^2+b^2}i-\frac{ab}{a^2+b^2}i-\frac{b^2}{a^2+b^2}i^2 = \frac{a^2}{a^2+b^2}+\frac{ab}{a^2+b^2}i-\frac{ab}{a^2+b^2}i+\frac{b^2}{a^2+b^2}$$

$$= \frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2} = \frac{a^2+b^2}{a^2+b^2} = 1$$

Since the product is 1, the multiplicative inverse is correct.

The final answer is $\frac{a}{a^2+b^2}+\frac{b}{a^2+b^2}i$.

Q4. Find the product of each complex number and its conjugate.

a. 4

Solution: 4

Identify the complex number:

The given complex number is 4. We can rewrite this as $4+0i$, where the real part is 4 and the imaginary part is 0.

Find the complex conjugate:

The complex conjugate of $4+0i$ is $4-0i$, which is simply 4.

Multiply the complex number by its conjugate:

Multiply 4 by its conjugate 4.

$\Rightarrow \quad 4 \cdot 4 = 16$

b. $1-i$

Solution: $1-i$

Identify the complex number:

The given complex number is $1-i$. Here, the real part is 1 and the imaginary part is -1.

Find the complex conjugate:

The complex conjugate of $1-i$ is $1+i$. We obtain this by changing the sign of the imaginary part.

Multiply the complex number by its conjugate:

Multiply $(1-i)$ by its conjugate $(1+i)$.

$\Rightarrow \quad (1-i)(1+i) = (1)^2-(i)^2 = 1-i^2$

Since $i^2 = -1$, we have:

$\Rightarrow \quad 1-(-1) = 1+1 = 2$

c. $7i$

Solution: $7i$

Find the complex conjugate:

The complex conjugate of a complex number $a+bi$ is $a-bi$.

We can rewrite $7i$ as $0 + 7i$.
Therefore, the complex conjugate of $7i$ is $0 - 7i$, which simplifies to $-7i$.
Multiply the complex number by its conjugate:

$\Rightarrow$ $(7i) \cdot (-7i) = -49i^2$

Simplify, remembering that $i^2 = -1$:

$\Rightarrow$ $-49i^2 = -49(-1) = 49$

Therefore, the product of the complex number $7i$ and its conjugate is 49.

d. $6 - 2i$

Solution: $6 - 2i$

Find the complex conjugate:
The complex conjugate of a complex number $a + bi$ is $a - bi$.
Therefore, the complex conjugate of $a - bi$ is $a + bi$.
The complex conjugate of $6 - 2i$ is $6 + 2i$.
Multiply the complex number by its conjugate:

$\Rightarrow$ $(6 - 2i)(6 + 2i) = (6)^2 - (2i)^2 = 36 - 4i^2$

Simplify, remembering that $i^2 = -1$:

$= 36 - 4(-1) = 36 + 4 = 40$

Therefore, the product of the complex number $6 - 2i$ and its conjugate is 40.

e. $10 + 9i$

Solution: $10 + 9i$

Find the complex conjugate:
The complex conjugate of a complex number $a + bi$ is $a - bi$.
Therefore, the complex conjugate of $10 + 9i$ is $10 - 9i$.
Multiply the complex number by its conjugate:

$\Rightarrow$ $(10 + 9i)(10 - 9i) = (10)^2 - (9i)^2 = 10 - 81i^2$

Simplify, remembering that $i^2 = -1$:

$= 100 - 81(-1) = 100 + 81 = 181$

Therefore, the product of the complex number $10 + 9i$ and its conjugate is 181.

f. $-4 - 11i$

Solution: $-4 - 11i$

Find the complex conjugate:
The complex conjugate of a complex number $a + bi$ is $a - bi$.
Therefore, the complex conjugate of $-4 - 11i$ is $-4 + 11i$.
Multiply the complex number by its conjugate:

$(-4 - 11i)(-4 + 11i) = (-4)^2 - (11i)^2 = 16 - 121i^2$

Simplify, remembering that $i^2 = -1$:

$= 16 - 121(-1) = 16 + 121 = 137$

Therefore, the product of the complex number $-4 - 11i$ and its conjugate is 137.

Q5. If $z_1 = 1 - 2i$ and $z_2 = 2 + i$:

a. **Show that:**

i. $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

Solution: $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

$\Rightarrow$ $z_1 = 1 - 2i$ and $z_2 = 2 + i$

- **Calculate $z_1 \cdot z_2$:**

$\Rightarrow$ $z_1 \cdot z_2 = (1 - 2i)(2 + i) = 1(2) + 1(i) - 2i(2) - 2i(i)$

$= 2 + i - 4i - 2i^2$

Since $i^2 = -1$, we have $2 + i - 4i + 2 = 4 - 3i$

- **Calculate $\overline{z_1 \cdot z_2}$:**

$\Rightarrow$ $\overline{z_1 \cdot z_2} = \overline{4 - 3i} = 4 + 3i$ ……… (1)

Now calculate $\overline{z_1}$ and $\overline{z_2}$.

$\Rightarrow$ $\overline{z_1} = \overline{1 - 2i} = 1 + 2i$

$\Rightarrow$ $\overline{z_2} = \overline{2 + i} = 2 - i$

- **Calculate $\overline{z_1} \cdot \overline{z_2}$:**

$\Rightarrow$ $\overline{z_1} \cdot \overline{z_2} = (1 + 2i)(2 - i) = 1(2) + 1(-i) + 2i(2) + 2i(-i)$

$= 2 - i + 4i - 2i^2$

Since $i^2 = -1$, we have $2 - i + 4i + 2 = 4 + 3i$ ……… (2)

Compare the results (1) and (2).

We found that $\overline{z_1 \cdot z_2} = 4 + 3i$ and $\overline{z_1} \cdot \overline{z_2} = 4 + 3i$.

Therefore, $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$ is verified.

ii. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

Solution: $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

$\Rightarrow$ $z_1 = 1 - 2i$ and $z_2 = 2 + i$

- **Calculate $\frac{z_1}{z_2}$:**

$\Rightarrow$ $\frac{z_1}{z_2} = \frac{1 - 2i}{2 + i}$ $\Rightarrow$ $\frac{z_1}{z_2} = \frac{1 - 2i}{2 + i} \times \frac{2 - i}{2 - i} = \frac{(1 - 2i)(2 - i)}{(2 + i)(2 - i)}$

- **Expanding the numerator:**

$\Rightarrow$ $(1 - 2i)(2 - i) = 1(2) + 1(-i) - 2i(2) - 2i(-i)$

$= 2 - i - 4i + 2i^2 = 2 - 5i - 2 = -5i$

- **Expanding the denominator:**

$\Rightarrow$ $(2 + i)(2 - i) = 2(2) + 2(-i) + i(2) + i(-i)$

$= 4 - 2i + 2i - i^2 = 4 + 1 = 5$

So, $\frac{z_1}{z_2} = \frac{-5i}{5} = -i$ ……… (1)

- **Calculate $\overline{\left(\frac{z_1}{z_2}\right)}$:**

$\Rightarrow$ $\overline{\left(\frac{z_1}{z_2}\right)} = \overline{-i} = i$

- **Calculate $\overline{z_1}$ and $\overline{z_2}$:**

$\Rightarrow$ $\overline{z_1} = \overline{1-2i} = 1+2i$ $\Rightarrow$ $\overline{z_2} = \overline{2+i} = 2-i$

- **Calculate $\frac{\overline{z_1}}{\overline{z_2}}$:**

$\Rightarrow$ $\frac{\overline{z_1}}{\overline{z_2}} = \frac{1+2i}{2-i}$ $\Rightarrow$ $\frac{1+2i}{2-i} \times \frac{2+i}{2+i} = \frac{(1+2i)(2+i)}{(2-i)(2+i)}$

- **Expanding the numerator:**

$\Rightarrow$ $(1+2i)(2+i) = 1(2) + 1(i) + 2i(2) + 2i(i)$

$= 2 + i + 4i + 2i^2 = 2 + 5i - 2 = 5i$

- **Expanding the denominator:**

$\Rightarrow$ $(2-i)(2+i) = 2(2) + 2(i) - i(2) - i(i)$

$= 4 + 2i - 2i - i^2 = 4 + 1 = 5$

So, $\frac{\overline{z_1}}{\overline{z_2}} = \frac{5i}{5} = i$ ……… (2)

Compare the results (1) and (2).

We found that $\overline{\left(\frac{z_1}{z_2}\right)} = i$ and $\frac{\overline{z_1}}{\overline{z_2}} = i$. Therefore, $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$ is verified.

iii. $|z_1| = |-z_1| = |\overline{z_1}| = |-\overline{z_1}|$

Solution: $z_1 = 1 - 2i$

- **Calculate $|z_1|$:**

$\Rightarrow$ $|z_1| = |1-2i| = \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$

- **Calculate $-z_1$:**

$\Rightarrow$ $-z_1 = -(1-2i) = -1+2i$

- **Calculate $|-z_1|$:**

$\Rightarrow$ $|-z_1| = |-1+2i| = \sqrt{(-1)^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$

- **Calculate $\overline{z_1}$:**

$\Rightarrow$ $\overline{z_1} = \overline{1-2i} = 1+2i$

- **Calculate $|\overline{z_1}|$:**

$\Rightarrow$ $|\overline{z_1}| = |1+2i| = \sqrt{1^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$

- **Calculate $-\overline{z_1}$:**

$\Rightarrow$ $-\overline{z_1} = -(1+2i) = -1-2i$

- **Calculate $|-\overline{z_1}|$:**

$\Rightarrow$ $|-\overline{z_1}| = |-1-2i| = \sqrt{(-1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$

Compare the results.

We found that $|z_1| = \sqrt{5}$, $|-z_1| = \sqrt{5}$, $|\overline{z_1}| = \sqrt{5}$ and $|-\overline{z_1}| = \sqrt{5}$.

Therefore, $|z_1| = |-z_1| = |\overline{z_1}| = |-\overline{z_1}|$ is verified.

iv. $z_2 . \overline{z_2} = |z_2|^2$

Solution: $z_2 = 2+i$

- **Calculate $\overline{z_2}$:**

$\Rightarrow$ $\overline{z_2} = \overline{2+i} = 2-i$

- **Calculate $z_2 \cdot \overline{z_2}$:**

$\Rightarrow$ $z_2 \cdot \overline{z_2} = (2+i)(2-i) = 2(2)+2(-i)+i(2)+i(-i)$
$= 4-2i+2i-i^2$

Since $i^2 = -1$, we have $4+1=5$

- **Calculate $|z_2|$:**

$\Rightarrow$ $|z_2| = |2+i| = \sqrt{2^2+1^2} = \sqrt{4+1} = \sqrt{5}$

- **Calculate $|z_2|^2$:**

$\Rightarrow$ $|z_2|^2 = (\sqrt{5})^2 = 5$

Compare the results.

We found that $z_2 \cdot \overline{z_2} = 5$ and $|z_2|^2 = 5$.

Therefore, $z_2 \cdot \overline{z_2} = |z_2|^2$ is verified.

b. **<u>Find:</u>**

i. $|z_1+z_2|$

Solution:

$\Rightarrow$ $z_1 = 1-2i$ and $z_2 = 2+i$

First, let's find the sum of z_1 and z_2:

$\Rightarrow$ $z_1+z_2 = (1-2i)+(2+i) = (1+2)+(-2i+i) = 3-i$

The magnitude of a complex number $a+bi$ is given by $\sqrt{a^2+b^2}$.

In this case, $a=3$ and $b=-1$.

$\Rightarrow$ $|z_1+z_2| = |3-i| = \sqrt{3^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

Thus, $|z_1+z_2| = \sqrt{10}$.

ii. $|z_1 z_2|$

Solution:

$\Rightarrow$ $z_1 = 1-2i$ and $z_2 = 2+i$

We can find the product $z_1 z_2$ first:

$\Rightarrow$ $z_1 z_2 = (1-2i)(2+i) = 1(2)+1(i)-2i(2)-2i(i)$
$= 2+i-4i-2i^2 = 2-3i-2(-1) = 2-3i+2$

Now we find the magnitude of $4-3i$:

$\Rightarrow$ $|z_1z_2| = |4-3i| = \sqrt{4^2+(-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

Alternatively, we can find the magnitudes of z_1 and z_2 separately and then multiply them.

$\Rightarrow$ $|z_1| = |1-2i| = \sqrt{1^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$

$\Rightarrow$ $|z_2| = |2+i| = \sqrt{2^2+1^2} = \sqrt{4+1} = \sqrt{5}$

Then, $|z_1z_2| = |z_1||z_2| = \sqrt{5}\cdot\sqrt{5} = 5.$

iii. $\left|\frac{z_1}{z_2}\right|$

Solution:

$\Rightarrow$ $z_1 = 1-2i$ and $z_2 = 2+i$

We can find the quotient $\frac{z_1}{z_2}$ first:

$\Rightarrow$ $\frac{z_1}{z_2} = \frac{1-2i}{2+i} = \frac{(1-2i)(2-i)}{(2+i)(2-i)} = \frac{2-i-4i+2i^2}{4-i^2} = \frac{2-5i-2}{4+1} = \frac{-5i}{5} = -i$

Now we find the magnitude of $-i$:

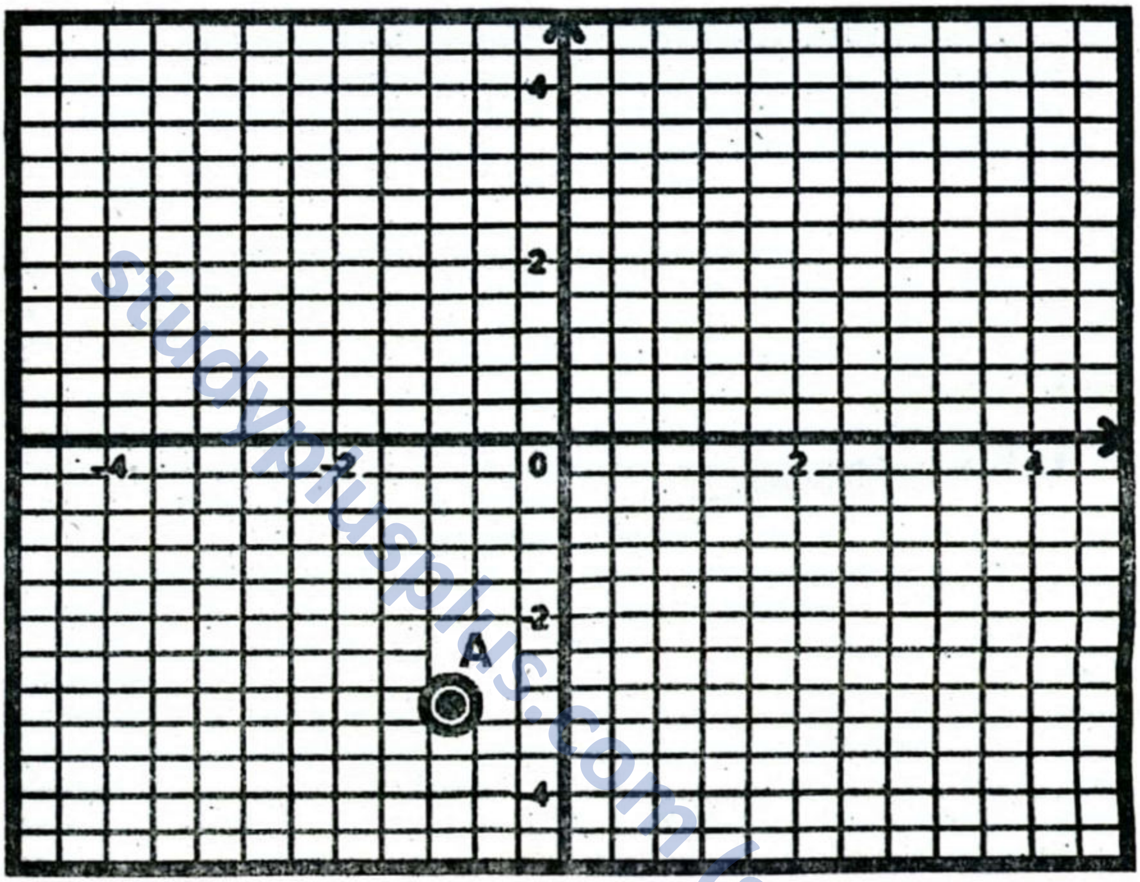
$\Rightarrow$ $\left|\frac{z_1}{z_2}\right| = |-i| = \sqrt{0^2+(-1)^2} = \sqrt{0+1} = \sqrt{1} = 1$

Q6. Represent the numbers in the complex plane.

a. $-1-3i$

Solution:

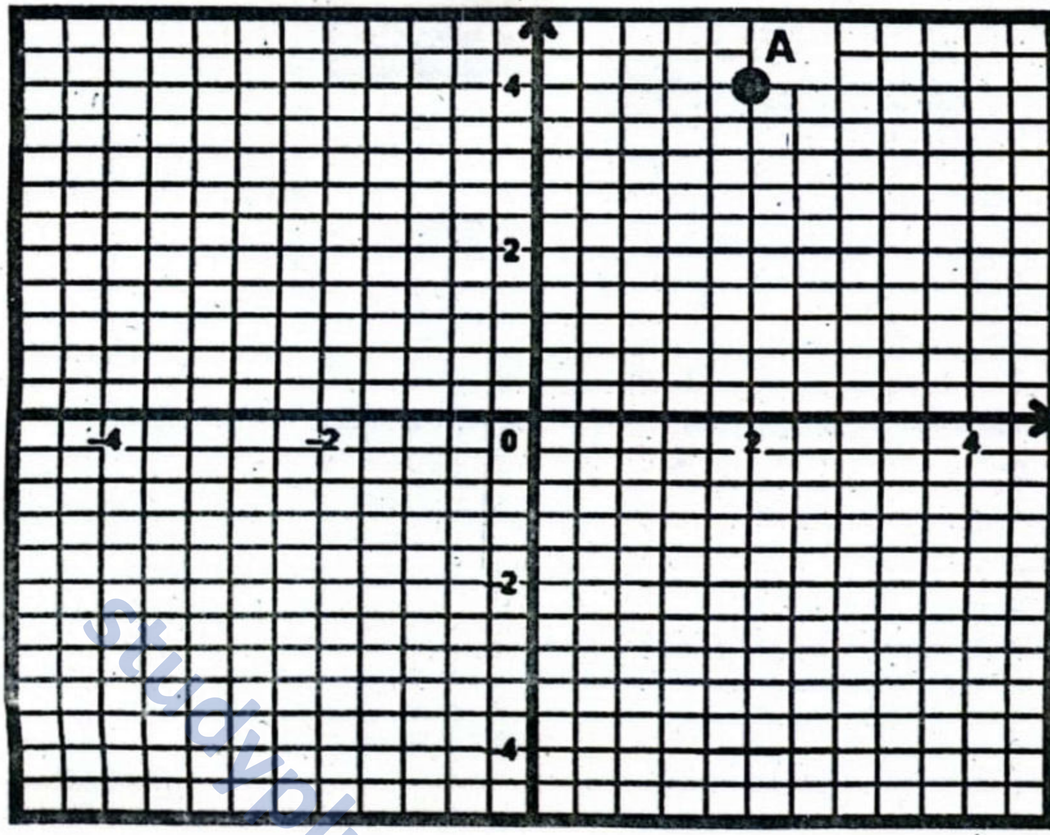
To represent the complex number $-1-3i$ in the complex plane, we plot the point with coordinates $A(-1,-3)$. The x-coordinate is the real part, and the y-coordinate is the imaginary part.



b. $2+4i$

Solution:

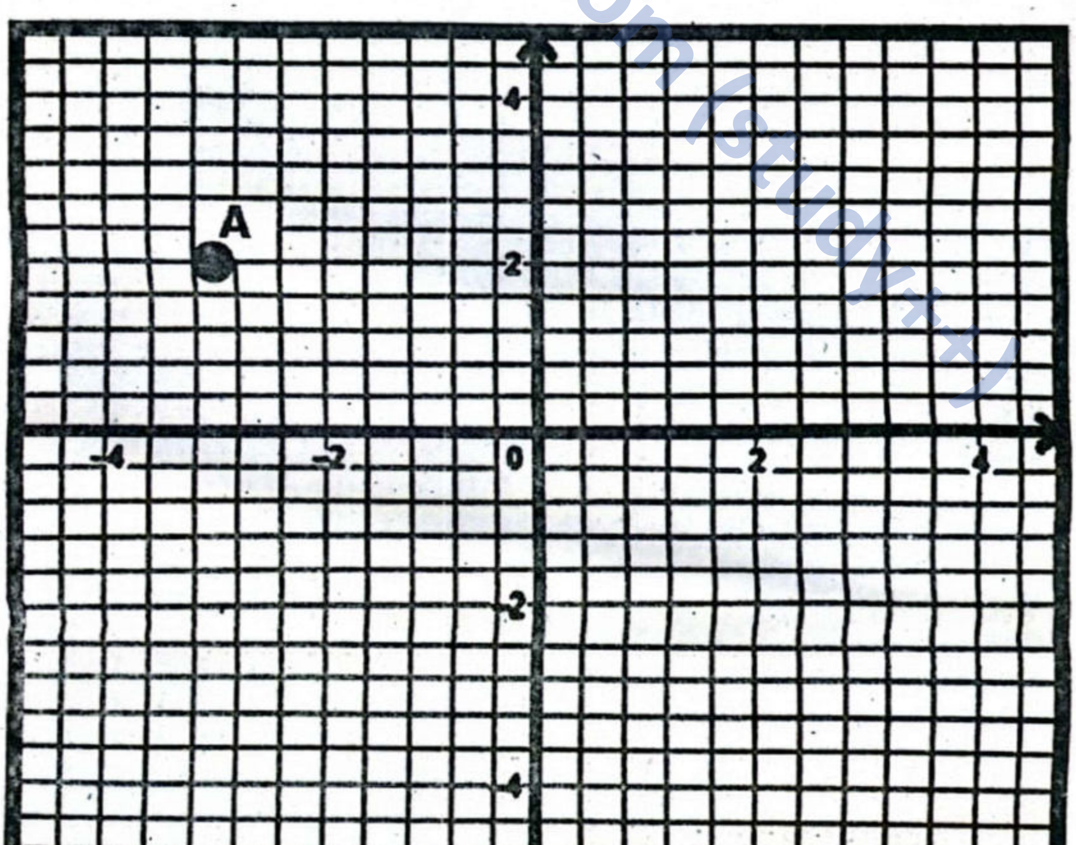
To represent the complex number $2+4i$ in the complex plane, we plot the point with coordinates $A(2,4)$. The x-coordinate is the real part, and the y-coordinate is the imaginary part.



c. $-3+2i$

Solution:

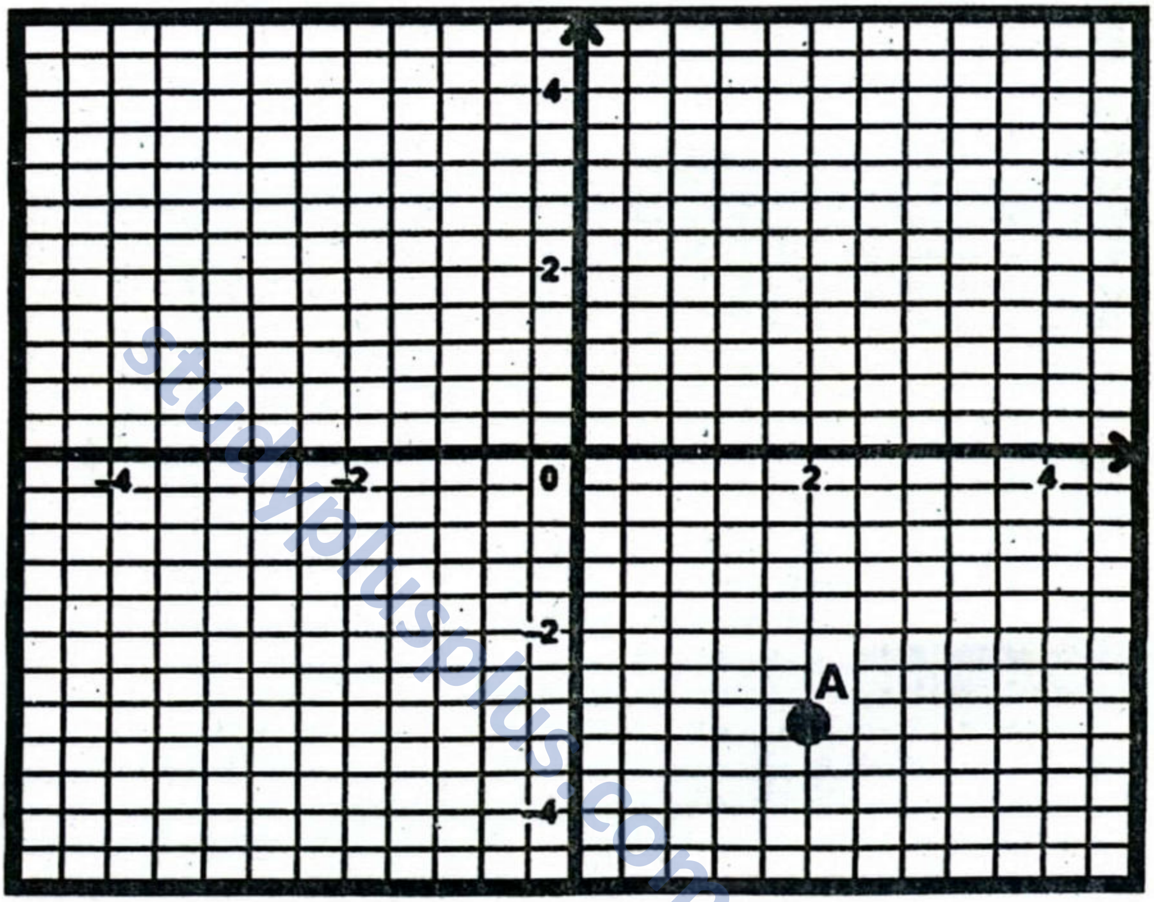
To represent the complex number $-3+2i$ in the complex plane, we plot the point with coordinates $\mathbf{A(-3,2)}$. The x-coordinate is the real part, and the y-coordinate is the imaginary part.



d. $2 - 3i$

Solution:

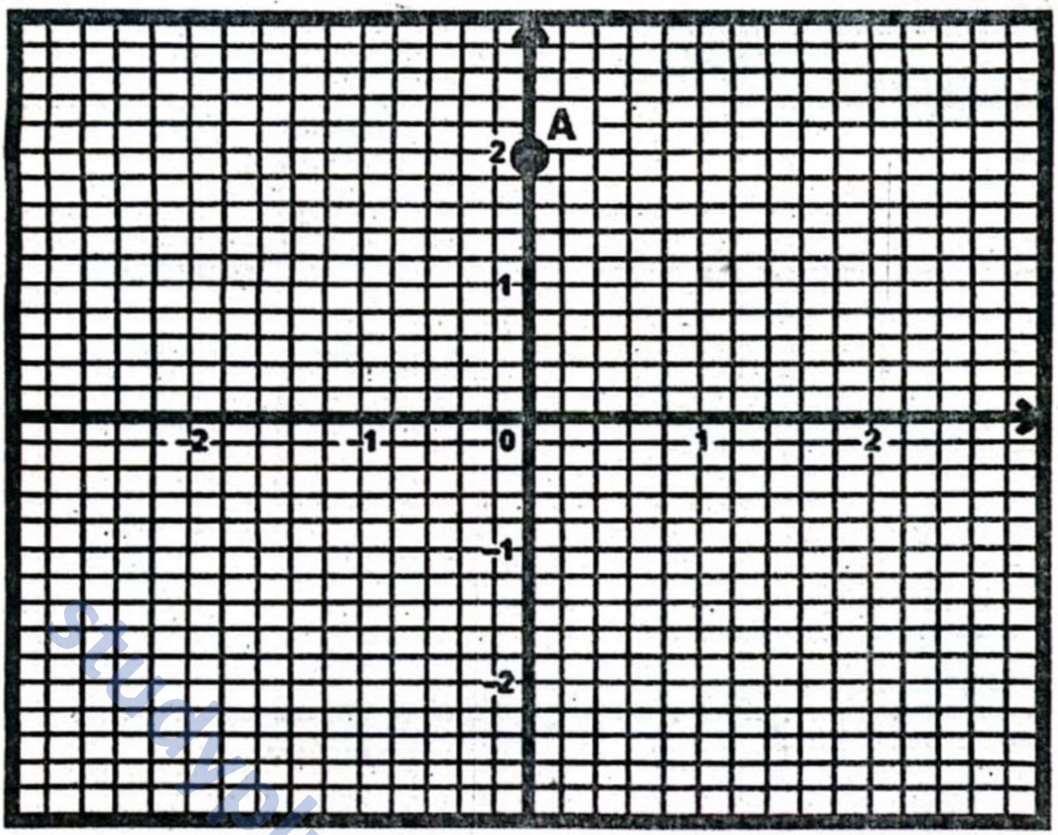
To represent the complex number $2 - 3i$ in the complex plane, we plot the point with coordinates $A(2, -3)$. The x-coordinate is the real part, and the y-coordinate is the imaginary part.



e. $2i$

Solution:

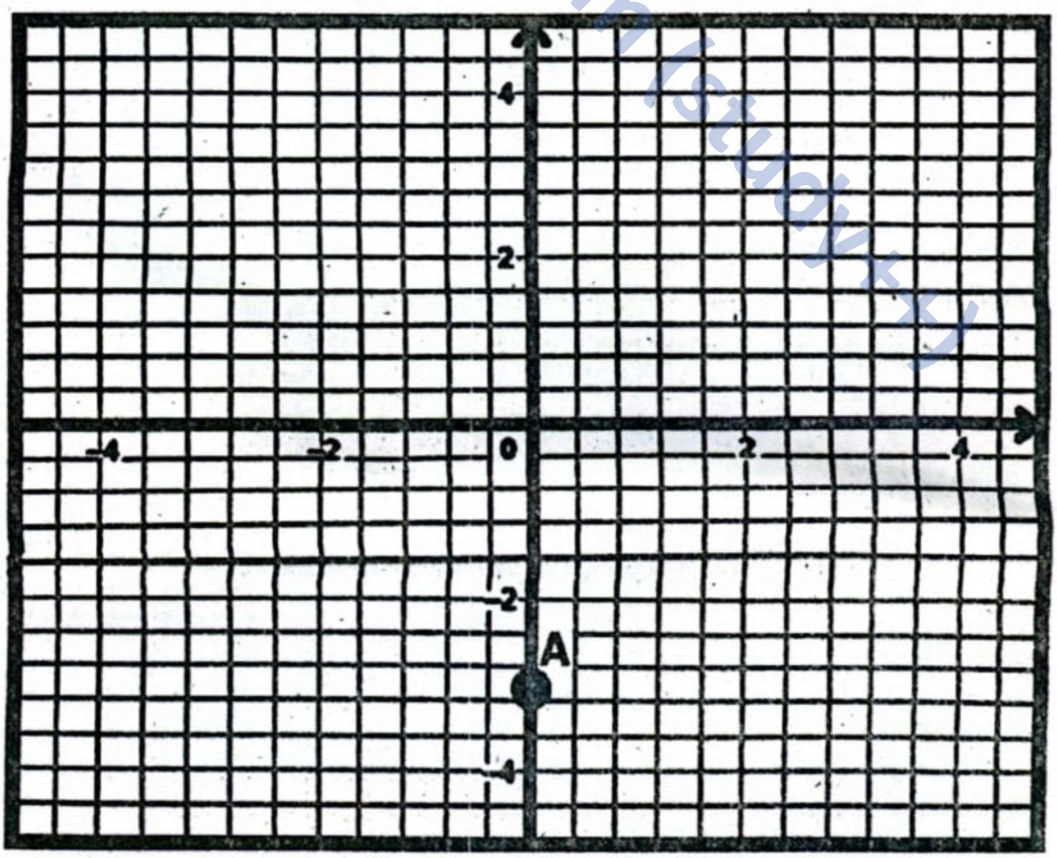
The complex number is $2i$, which can be written as $0 + 2i$. To graph the complex number $2i$ in the complex plane, we plot the point $A(0, 2)$, where the x-axis represents the real part and the y-axis represents the imaginary part.



f. $-3i$

Solution:

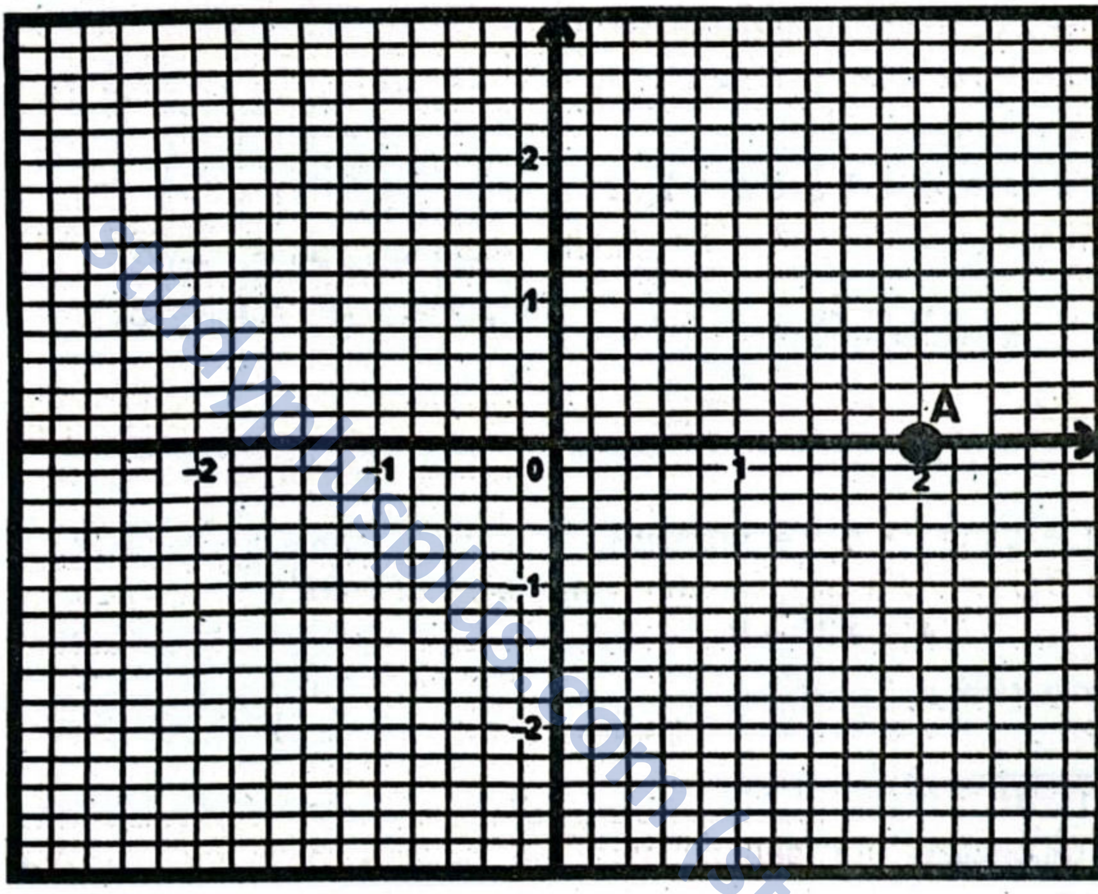
The complex number is $-3i$, which can be written as $0-3i$. To graph the complex number $-3i$ in the complex plane, we plot the point $A(0,-3)$, where the x-axis represents the real part and the y-axis represents the imaginary part.



g. 2

Solution:

To represent the number 2 in the complex plane, we consider it as a complex number $z = 2 + 0i$. The complex plane has a real axis (horizontal) and an imaginary axis (vertical). The complex number $z = a + bi$ is represented by the point (a, b) in the complex plane. In our case, $z = 2 + 0i$, so $a = 2$ and $b = 0$. The point representing this complex number is $A(2, 0)$. This point lies on the real axis, 2 units away from the origin.



Q7. **Separate into real and imaginary parts of each complex number.**

a. $(\sqrt{2} - \sqrt{3}i)^2$

Solution: $(\sqrt{2} - \sqrt{3}i)^2$

$\Rightarrow$ $(\sqrt{2} - \sqrt{3}i)^2 = (\sqrt{2} - \sqrt{3}i)(\sqrt{2} - \sqrt{3}i)$

$= (\sqrt{2})(\sqrt{2}) + (\sqrt{2})(-\sqrt{3}i) + (-\sqrt{3}i)(\sqrt{2}) + (-\sqrt{3}i)(-\sqrt{3}i)$

$= 2 - \sqrt{6}i - \sqrt{6}i + 3i^2$

$= 2 - 2\sqrt{6}i - 3 = -1 - 2\sqrt{6}i$

The complex number is now in the form $a + bi$, where a is the real part and b is the imaginary part.

In this case, $a = -1$ and $b = -2\sqrt{6}$.

Therefore, the real part is -1 and the imaginary part is $-2\sqrt{6}$.

Real part $= R_e = -1$; Imaginary part $= I_m = -2\sqrt{6}$

b. $(\sqrt{2}+i)^2$

Solution: $(\sqrt{2}+i)^2$

$\Rightarrow$ $(\sqrt{2}+i)^2 = (\sqrt{2}+i)(\sqrt{2}+i)$

$= (\sqrt{2})(\sqrt{2}) + (\sqrt{2})(i) + (\sqrt{2})(i) + (i)(i)$

$= 2 + \sqrt{2}i + \sqrt{2}i + i^2$

$= 2 - 1 + 2\sqrt{2}i$; $[\because \ i^2 = -1]$

$= 1 + 2\sqrt{2}i$

The complex number is now in the form $a+bi$, where a is the real part and b is the imaginary part.

In this case, $a = 1$ and $b = 2\sqrt{2}$.

Therefore, the real part is -1 and the imaginary part is $-2\sqrt{6}$.

Real part $= R_e = 1$; Imaginary part $= I_m = 2\sqrt{2}$

c. $\dfrac{(2+3i)^2}{1-3i}$

Solution:

First, expand the numerator:

$\Rightarrow$ $(2+3i)^2 = (2+3i)(2+3i) = 4 + 6i + 6i + 9i^2$

$= 4 + 12i - 9 = -5 + 12i$

Now, we have:

$= \dfrac{-5+12i}{1-3i}$

To rationalize the denominator, we multiply both the numerator and denominator by the conjugate of the denominator, which is $1+3i$.

$\Rightarrow$ $\dfrac{-5+12i}{1-3i} \times \dfrac{1+3i}{1+3i} = \dfrac{(-5+12i)(1+3i)}{(1-3i)(1+3i)}$

$= \dfrac{-5-15i+12i+36i^2}{1+3i-3i-9i^2} = \dfrac{-5-15i+12i+36i^2}{1+3i-3i-9i^2}$; $[\because \ i^2 = -1]$

$= \dfrac{-5-3i-36}{1+9} = \dfrac{-41-3i}{10} = -\dfrac{41}{10} - \dfrac{3}{10}i$

The complex number is now in the form $a+bi$, where a is the real part and b is the imaginary part.

In this case, $a = -\dfrac{41}{10}$ and $b = -\dfrac{3}{10}$.

Therefore, the real part is $-\dfrac{41}{10}$ and the imaginary part is $-\dfrac{3}{10}$.

Real part $= R_e = -\dfrac{41}{10}$; Imaginary part $= I_m = -\dfrac{3}{10}$

d. $\left[\frac{1}{2}+\frac{\sqrt{3}}{2}i\right]^2$

Solution: $\left[\frac{1}{2}+\frac{\sqrt{3}}{2}i\right]^2$

$$\Rightarrow \left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^2 = \left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right) + \left(\frac{\sqrt{3}}{2}i\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}i\right)\left(\frac{\sqrt{3}}{2}i\right)$$

$$= \frac{1}{4}+\frac{\sqrt{3}}{4}i+\frac{\sqrt{3}}{4}i+\frac{3}{4}i^2 = \frac{1}{4}-\frac{3}{4}+\frac{2\sqrt{3}}{4}i \;; \qquad [\because i^2=-1]$$

$$= -\frac{2}{4}+\frac{\sqrt{3}}{2}i = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

The complex number is now in the form $a+bi$, where a is the real part and b is the imaginary part.

In this case, $a=-\frac{1}{2}$ and $b=\frac{\sqrt{3}}{2}$.

Therefore, the real part is $-\frac{1}{2}$ and the imaginary part is $\frac{\sqrt{3}}{2}$.

Real part $= R_e = -\frac{1}{2}$; Imaginary part $= I_m = \frac{\sqrt{3}}{2}$

e. $\frac{1-i}{(i)^2}$

Solution: $\frac{1-i}{(i)^2}$

We know that $i^2=-1$, so we have:

$$\Rightarrow \frac{1-i}{(i)^2} = \frac{1-i}{-1} \;; \qquad [\because i^2=-1]$$

Now, we can divide both the real and imaginary parts of the numerator by -1.

$$\Rightarrow \frac{1}{-1}-\frac{i}{-1} = -1+i$$

The complex number is now in the form $a+bi$, where a is the real part and b is the imaginary part.

In this case, $a=-1$ and $b=1$.

Therefore, the real part is -1 and the imaginary part is 1.

Real part $= R_e = -1$; Imaginary part $= 1$

f. $\frac{1}{i(1-i)^2}$

Solution: $\frac{1}{i(1-i)^2}$

First, expand $(1-i)^2$:

$\Rightarrow$ $(1-i)^2 = (1-i)(1-i) = 1 - i - i + i^2$

$= 1 - 2i - 1 = -2i$; $[\because \; i^2 = -1]$

Now, we have:

$\Rightarrow$ $\frac{1}{i(-2i)} = \frac{1}{-2i^2} = \frac{1}{-2(-1)} = \frac{1}{2}$

The complex number is now in the form $a + bi$, where a is the real part and b is the imaginary part.

In this case, $a = \frac{1}{2}$ and $b = 0$.

Therefore, the real part is $\frac{1}{2}$ and the imaginary part is 0.

Real part $= R_e = \frac{1}{2}$; Imaginary part $= 0$

g. $\frac{(1+i)^2}{(1-2i)^2}$

Solution: $\frac{(1+i)^2}{(1-2i)^2}$

Expand the numerator and denominator:

$\Rightarrow$ $(1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$; $[\because \; i^2 = -1]$

$\Rightarrow$ $(1-2i)^2 = 1 - 4i + (2i)^2 = 1 - 4i + 4i^2 = 1 - 4i - 4 = -3 - 4i$

So, we have: $\frac{(1+i)^2}{(1-2i)^2} = \frac{2i}{-3-4i}$

To get rid of the imaginary part in the denominator, multiply both the numerator and denominator by the conjugate of $-3-4i$, which is $-3+4i$.

$\Rightarrow$ $\frac{2i}{-3-4i} \times \frac{-3+4i}{-3+4i} = \frac{2i(-3+4i)}{(-3-4i)(-3+4i)}$

Simplify.

Numerator:

$\Rightarrow$ $2i(-3+4i) = -6i + 8i^2 = -6i - 8 = -8 - 6i$

Denominator:

$\Rightarrow$ $(-3-4i)(-3+4i) = (-3)^2 - (4i)^2 = 9 - 16i^2 = 9 + 16 = 25$

So, we have: $= \frac{-8-6i}{25}$

Separate into real and imaginary parts:

$\Rightarrow$ $\frac{-8-6i}{25} = \frac{-8}{25} - \frac{6}{25}i$

Real part $= R_e = -\frac{8}{25}$; Imaginary part $= -\frac{6}{25}$

Q8. Taking any complex number and show that:

a. $z.\bar{z}$ **is a real number.**

Solution:

Let $z = a + bi$ be any complex number, where a and b are real numbers.

Then the complex conjugate of z is $\bar{z} = a - bi$.

Now, let's compute the product of z and $\bar{z}$.

$\Rightarrow \quad z \cdot \bar{z} = (a + bi)(a - bi)$

Using the difference of squares formula, we have:

$\Rightarrow \quad z \cdot \bar{z} = a^2 - (bi)^2 = a^2 - b^2 i^2$

Since $i^2 = -1$, we get:

$\Rightarrow \quad z \cdot \bar{z} = a^2 - b^2(-1) = a^2 + b^2$

Since a and b are real numbers, a^2 and b^2 are also real numbers.

Therefore, $a^2 + b^2$ is a real number.

Thus, we have shown that the product of a complex number z and its complex conjugate $\bar{z}$ is a real number.

$\Rightarrow \quad z \cdot \bar{z} = a^2 + b^2$

This is also equal to the square of the magnitude of z, i.e., $|z|^2 = a^2 + b^2$.

b. $z^2 + (\bar{z})^2$ **is a real number.**

Solution:

Let $z = a + bi$ be a complex number, where a and b are real numbers.

Then the complex conjugate of z is $\bar{z} = a - bi$.

We want to show that $z^2 + (\bar{z})^2$ is a real number.

First, we compute z^2.

$\Rightarrow \quad z^2 = (a + bi)^2 = a^2 + 2abi + (bi)^2 = a^2 + 2abi - b^2$

$= (a^2 - b^2) + 2abi$

Next, we compute $(\bar{z})^2$.

$\Rightarrow \quad (\bar{z})^2 = (a - bi)^2 = a^2 - 2abi + (bi)^2 = a^2 - 2abi - b^2$

$= (a^2 - b^2) - 2abi$

Now, we add z^2 and $(\bar{z})^2$.

$\Rightarrow \quad z^2 + (\bar{z})^2 = [(a^2 - b^2) + 2abi] + [(a^2 - b^2) - 2abi]$

$= (a^2 - b^2) + (a^2 - b^2) + 2abi - 2abiz^2 + (\bar{z})^2 = 2(a^2 - b^2)$

Since a and b are real numbers, a^2 and b^2 are also real numbers.

Therefore, $a^2 - b^2$ is a real number, and $2(a^2 - b^2)$ is also a real number.

Thus, we have shown t $z^2 + (\bar{z})^2$ is a real number.

c. $(z - \bar{z})^2$ **is a real number.**

Solution:

Let $z = a + bi$ be a complex number, where a and b are real numbers.

Then the complex conjugate of z is $\bar{z} = a - bi$.

We want to show that $(z - \bar{z})^2$ is a real number.

First, we compute $-\bar{z}$.

$\Rightarrow \quad z - \bar{z} = (a + bi) - (a - bi) = a + bi - a + bi = 2bi$

Now, we compute $(z-\bar{z})^2$.

$\Rightarrow \quad (z-\bar{z})^2 = (2bi)^2 = (2b)^2 i^2 = 4b^2 i^2$

Since $i^2 = -1$, we have:

$\Rightarrow \quad (z-\bar{z})^2 = 4b^2(-1) = -4b^2$

Since b is a real number, b^2 is also a real number.

Therefore, $-4b^2$ is a real number.

Thus, we have shown that $(z-\bar{z})^2$ is a real number.

d. $|z|$ and $|\bar{z}|$ are real numbers.

Solution:

Let $z = a + bi$ be a complex number, where a and b are real numbers.

Then the complex conjugate of z is $\bar{z} = a - bi$.

The magnitude of z, denoted by $|z|$, is defined as: $z = a + bi$.

$\Rightarrow \quad |z| = \sqrt{a^2 + b^2}$

Since a and b are real numbers, a^2 and b^2 are also real numbers. The sum of two real numbers is a real number, so $a^2 + b^2$ is a real number. The square root of a non-negative real number is also a real number.

Therefore, $|z| = \sqrt{a^2 + b^2}$ is a real number.

The magnitude of $\bar{z}$, denoted by $|\bar{z}|$, is defined as: $\bar{z} = a - bi$

$\Rightarrow \quad |\bar{z}| = \sqrt{a^2 + (-b)^2} = \sqrt{a^2 + b^2}$

Since a and b are real numbers, a^2 and b^2 are also real numbers. The sum of two real numbers is a real number, so $a^2 + b^2$ is a real number. The square root of a non-negative real number is also a real number.

Therefore, $|\bar{z}| = \sqrt{a^2 + b^2}$ is a real number.

Note that $|z| = |\bar{z}|$.

Thus, we have shown that $|z|$ and $|\bar{z}|$ are real numbers.

e. $z^2 - (\bar{z})^2$ is an imaginary number.

Solution:

Let $z = a + bi$ be a complex number, where a and b are real numbers.

Then the complex conjugate of z is $\bar{z} = a - bi$.

We want to show that $z^2 - (\bar{z})^2$ is an imaginary number.

First, we compute z^2.

$\Rightarrow \quad z^2 = (a+bi)^2 = a^2 + 2abi + (bi)^2 = a^2 + 2abi - b^2$

$= (a^2 - b^2) + 2abi$

Next, we compute $(\bar{z})^2$.

$\Rightarrow \quad (\bar{z})^2 = (a-bi)^2 = a^2 - 2abi + (bi)^2 = a^2 - 2abi - b^2$

$= (a^2 - b^2) - 2abi$

Now, we subtract $(\bar{z})^2$ from z^2.

$\Rightarrow \quad z^2 - (\bar{z})^2 = [(a^2 - b^2) + 2abi] - [(a^2 - b^2) - 2abi]$

$\Rightarrow \quad z^2 - (\bar{z})^2 = (a^2 - b^2) - (a^2 - b^2) + 2abi - (-2abi)$

$\Rightarrow \quad z^2 - (\bar{z})^2 = a^2 - b^2 - a^2 + b^2 + 2abi + 2abi = 4abi$

The result is $4abi$. This is an imaginary number because it is a real number ($4ab$) multiplied by the imaginary unit i. If $a = 0$ or $b = 0$, then $4abi = 0$, which is a real number. However, if $a \neq 0$ and $b \neq 0$, then $4abi$ is a purely imaginary number.

Thus, $z^2 - (\bar{z})^2$ is an imaginary number.

Q9. <u>Represent sum and difference of complex numbers graphically.</u>

(i) $z_1 = 5 + 3i$ **and** $z_2 = 2 - 3i$

Solution:

Given:

$z_1 = 5 + 3i = (5, 3)$; $z_2 = 2 - 3i = (2, -3)$

First, let's find the sum $z_1 + z_2$.

$\Rightarrow$ $z_1 + z_2 = (5 + 3i) + (2 - 3i) = (5 + 2) + (3i - 3i) = 7 + 0i = (7, 0)$

Next, let's find the difference $z_1 - z_2$.

$\Rightarrow$ $z_1 - z_2 = (5 + 3i) - (2 - 3i) = (5 - 2) + [3i - (-3i)] = 3 + 6i$

- **<u>Graphical Representation:</u>**

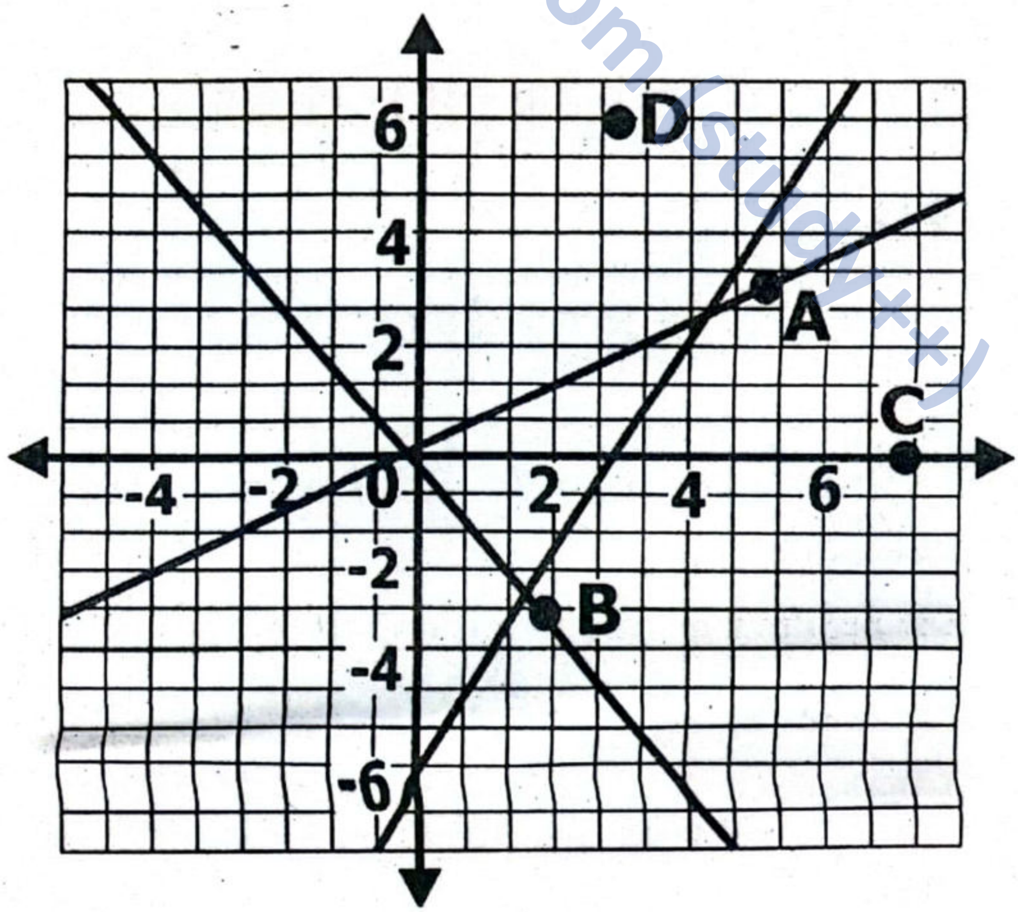
Now, let's illustrate these operations graphically using vectors in the complex plane. We will represent each complex number as a vector from the origin to the point corresponding to the complex number.

$\Rightarrow$ $z_1 = 5 + 3i$ is represented by the vector from $O(0, 0)$ to $A(5, 3)$.

$\Rightarrow$ $z_2 = 2 - 3i$ is represented by the vector from $O(0, 0)$ to $B(2, -3)$.

$\Rightarrow$ $z_1 + z_2 = 7$ is represented by the vector from $O(0, 0)$ to $C(7, 0)$.

$\Rightarrow$ $z_1 - z_2 = 3 + 6i$ is represented by the vector from $O(0, 0)$ to $D(3, 6)$.



(ii) $z_1 = -3 + 2i$ **and** $z_2 = 4 + 3i$

Solution:

Given:

$z_1 = -3 + 2i$; $z_2 = 4 + 3i$

First, let's find the sum $z_1 + z_2$.

$z_1 + z_2 = (-3 + 2i) + (4 + 3i) = (-3 + 4) + (2i + 3i) = 1 + 5i$

Next, let's find the difference $z_1 - z_2$.

$z_1 - z_2 = (-3 + 2i) - (4 + 3i) = (-3 - 4) + (2i - 3i) = -7 - i$

- **Graphical Representation:**

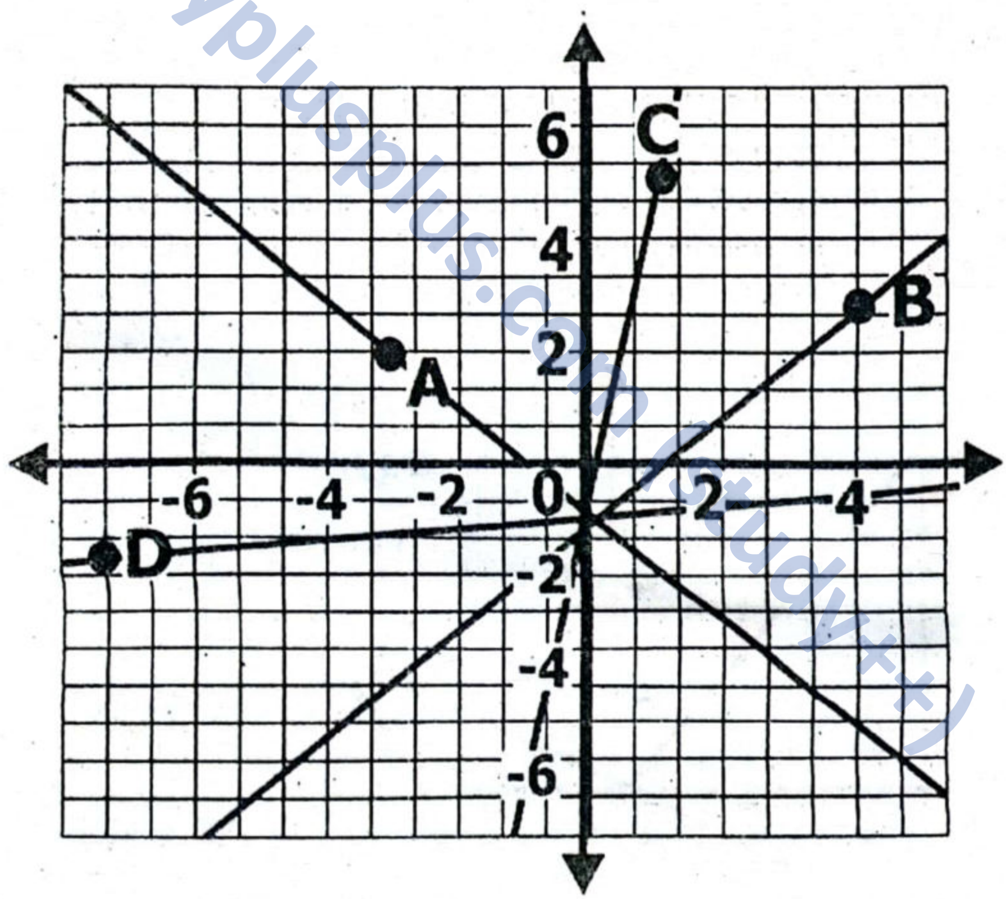
Now, let's illustrate these operations graphically using vectors in the complex plane. We will represent each complex number as a vector from the origin to the point corresponding to the complex number.

$\Longrightarrow$ $z_1 = -3 + 2i$ is represented by the vector from $O(0, 0)$ to $A(-3, 2)$.

$\Longrightarrow$ $z_2 = 4 + 3i$ is represented by the vector from $O(0, 0)$ to $B(4, 3)$.

$\Longrightarrow$ $z_1 + z_2 = 1 + 5i$ is represented by the vector from $O(0, 0)$ to $C(1, 5)$.

$\Longrightarrow$ $z_1 - z_2 = -7 - i$ is represented by the vector from $O(0, 0)$ to $D(-7, -1)$.



Q10. Represent product of complex numbers graphically.

(i) $z_1 = 4 + 2i$ **and** $z_2 = -2 + 3i$

Solution:

First, we need to calculate the product $z_1 z_2$ where $z_1 = 4 + 2i$ and $z_2 = -2 + 3i$.

$\Rightarrow \quad z_1 z_2 = (4+2i)(-2+3i)$

$= 4(-2) + 4(3i) + 2i(-2) + 2i(3i) = -8 + 12i - 4i + 6i^2$

Since $i^2 = -1$, we have:

$\Rightarrow \quad z_1 z_2 = -8 + 8i - 6 = -14 + 8i$

Now, we need to express z_1, z_2 and $z_1 z_2$ in polar form. The polar form of a complex number $z = a + bi$ is given by $z = r(\cos\theta + i\sin\theta)$, where:

$\Rightarrow \quad r = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

- **For $z_1 = 4 + 2i$**
  - $r_1 = \sqrt{4^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$
  - $\theta_1 = \tan^{-1}\left(\frac{2}{4}\right) = \tan^{-1}\left(\frac{1}{2}\right) \approx 26.57^\circ$

  So, $z_1 = 2\sqrt{5}\,(\cos 27^\circ + i\sin 27^\circ)$ (rounded to the nearest degree)

- **For $z_2 = -2 + 3i$**
  - $r_2 = \sqrt{(-2)^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13}$
  - $\theta_2 = \tan^{-1}\left(\frac{3}{-2}\right)$

  Since the complex number is in the second quadrant, we add 180° to the result.

  $\Rightarrow \quad \theta_2 = \tan^{-1}(-1.5) \approx -56.31^\circ$

  So, $\theta_2 = -56.31^\circ + 180^\circ \approx 123.69^\circ$

  So, $z_2 = \sqrt{13}\,(\cos 124^\circ + i\sin 124^\circ)$ (rounded to the nearest degree)

- **For $z_1 z_2 = -14 + 8i$**
  - $r_3 = \sqrt{(-14)^2 + 8^2} = \sqrt{196 + 64} = \sqrt{260} = 2\sqrt{65}$
  - $\theta_3 = \tan^{-1}\left(\frac{8}{-14}\right)$

  Since the complex number is in the second quadrant, we add 180° to the result.

  $\Rightarrow \quad \theta_3 = \tan^{-1}\left(-\frac{4}{7}\right) \approx -29.74^\circ$

  So, $\theta_3 = -29.74^\circ + 180^\circ \approx 150.26^\circ$

  So, $z_1 z_2 = 2\sqrt{65}\,(\cos 150^\circ + i\sin 150^\circ)$ (rounded to nearest degree)

Therefore,

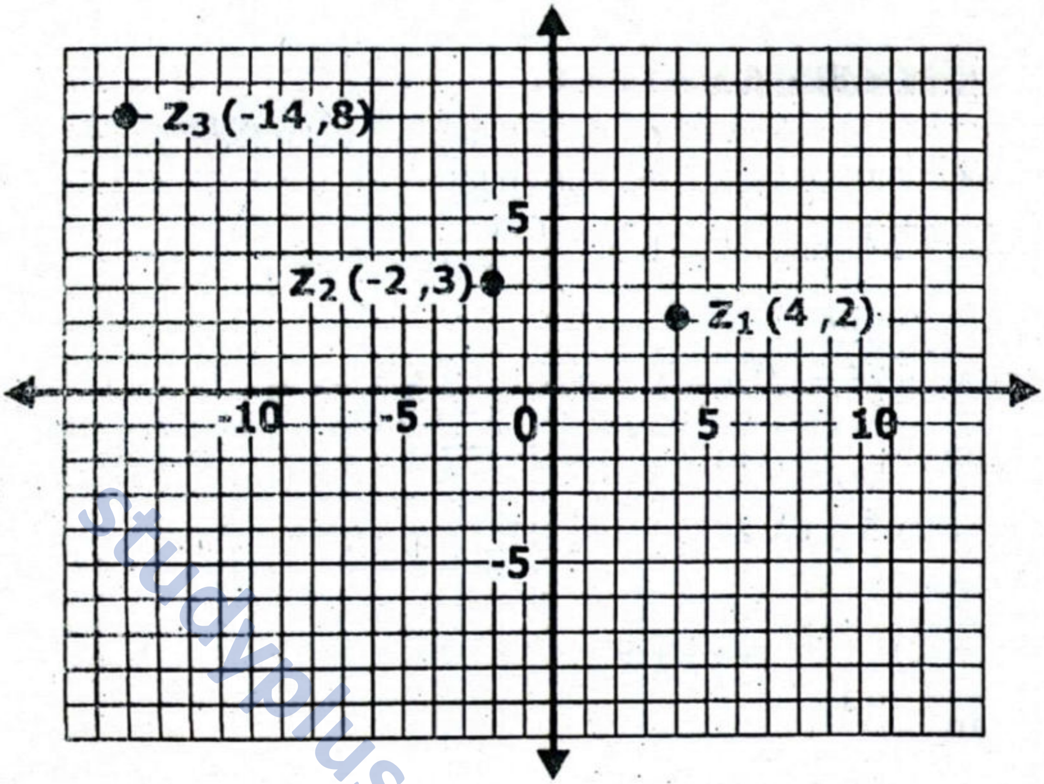
$\Rightarrow \quad z_1 = 2\sqrt{5}\,(\cos 27^\circ + i\sin 27^\circ)$

$\Rightarrow \quad z_2 = \sqrt{13}\,(\cos 124^\circ + i\sin 124^\circ)$

$\Rightarrow \quad z_1 z_2 = 2\sqrt{65}\,(\cos 150^\circ + i\sin 150^\circ)$

- **Graphical Representation:**

Now, let's represent these complex numbers graphically.



$\Rightarrow \quad z_1 = 4 + 2i = (4, 2)\ ; \quad z_2 = -2 + 3i\ (-2, 3)$

$\Rightarrow \quad z_3 = z_1 z_2 = -14 + 8i = (-14, 8)$

(ii) $z_1 = -2 + 4i$ **and** $z_2 = 3 - i$

Solution:

First, we need to calculate the product $z_1 z_2$ where $z_1 = -2 + 4i$ and $z_2 = 3 - i$.

$\Rightarrow \quad z_1 z_2 = (-2 + 4i)(3 - i)$

$$= -2(3) - 2(-i) + 4i(3) + 4i(-i) = -6 + 2i + 12i - 4i^2$$

Since $i^2 = -1$, we have:

$\Rightarrow \quad z_1 z_2 = -6 + 14i + 4 = -2 + 14i$

Now, we need to express z_1, z_2, and $z_1 z_2$ in polar form. The polar form of a complex number $z = a + bi$ is given by $z = r(\cos\theta + i\sin\theta)$, where:

$\Rightarrow \quad r = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

➢ **For** $z_1 = -2 + 4i$

- $r_1 = \sqrt{(-2)^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$
- $\theta_1 = \tan^{-1}\left(\frac{4}{-2}\right) = \tan^{-1}(-2)$

Since the complex number is in the second quadrant, we add 180° to the result.

$\Rightarrow \quad \theta_1 \approx -63.43^\circ + 180^\circ \approx 116.57^\circ$

So, $z_1 = 2\sqrt{5}\,(\cos 117^\circ + i \sin 117^\circ)$ (rounded to the nearest degree)

- **For $z_2 = 3 - i$**
- $r_2 = \sqrt{3^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$
- $\theta_2 = \tan^{-1}\left(\frac{-1}{3}\right) \approx -18.43^\circ$

Since the complex number is in the fourth quadrant, we can also express this as $360^\circ - 18.43^\circ \approx 341.57^\circ$.

So, $z_2 = \sqrt{10}\,(\cos 342^\circ + i \sin 342^\circ)$ (rounded to the nearest degree)

- **For $z_1\, z_2 = -2 + 14i$**
- $r_3 = \sqrt{(-2)^2 + 14^2} = \sqrt{4+196} = \sqrt{200} = 10\sqrt{2}$
- $\theta_3 = \tan^{-1}\left(\frac{14}{-2}\right) = \tan^{-1}(-7)$

Since the complex number is in the second quadrant, we add 180° to the result.

$\Rightarrow \theta_3 \approx -81.87^\circ + 180^\circ \approx 98.13^\circ$

So, $z_1\, z_2 = 10\sqrt{2}\,(\cos 98^\circ + i \sin 98^\circ)$ (rounded to the nearest degree)

Therefore,

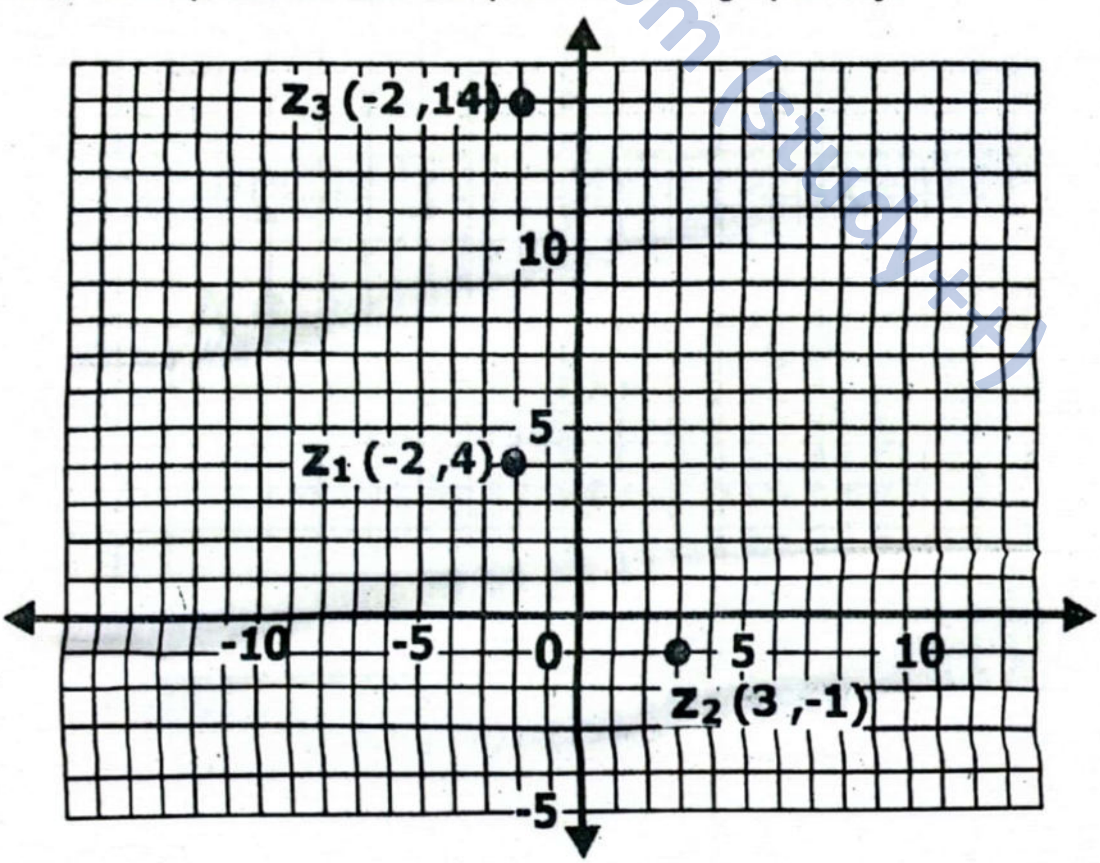
$\Rightarrow z_1 = 2\sqrt{5}\,(\cos 117^\circ + i \sin 117^\circ)$

$\Rightarrow z_2 = \sqrt{10}\,(\cos 342^\circ + i \sin 342^\circ)$

$\Rightarrow z_1 z_2 = 10\sqrt{2}\,(\cos 98^\circ + i \sin 98^\circ)$

- **<u>Graphical Representation:</u>**

Now, let's represent these complex numbers graphically.



$\Rightarrow \quad z_1 = -2 + 4i = (-2, 4)\ ;\ \ z_2 = 3 - i\ (3, -1)$

$\Rightarrow \quad z_3 = z_1 z_2 = -2 + 14i = (-2, 14)$

Q11. Represent $z_1 \div z_2$ graphically when:

(i) $z_1 = 6 - 4i$ **and** $z_2 = 3$

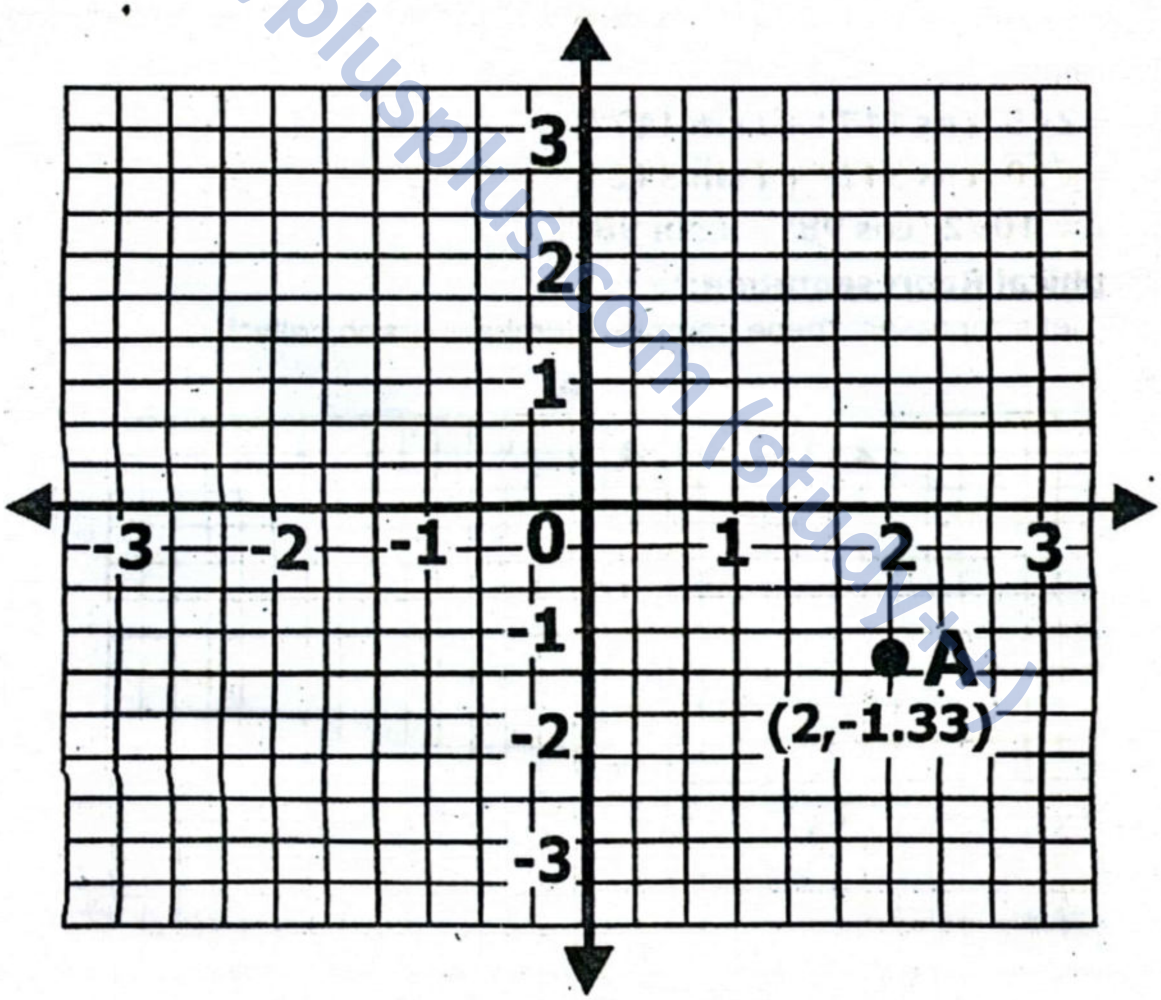
Solution:

We are asked to represent $\frac{z_1}{z_2}$ graphically when $z_1 = 6 - 4i$ and $z_2 = 3$.

First, we compute $\frac{z_1}{z_2}$:

$$\Rightarrow \quad \frac{z_1}{z_2} = \frac{6-4i}{3} = \frac{6}{3} - \frac{4}{3}i = 2 - \frac{4}{3}i$$

To represent this complex number graphically, we plot the point $\left(2, -\frac{4}{3}\right)$ on the complex plane, where the $x-$axis represents the real part and the $y-$axis represents the imaginary part.



(ii) $z_1 = -4 - 6i$ **and** $z_2 = 1 + i$

Solution:

We are asked to represent $\frac{z_1}{z_2}$ graphically when $z_1 = -4 - 6i$ & $z_2 = 1 + i$.

First, we compute $\frac{z_1}{z_2}$:

$$\Rightarrow \quad \frac{z_1}{z_2} = \frac{-4-6i}{1+i}$$

To divide complex numbers, we multiply the numerator and denominator by the conjugate of the denominator:

$$\Rightarrow \quad \frac{-4-6i}{1+i} = \frac{(-4-6i)(1-i)}{(1+i)(1-i)} = \frac{-4+4i-6i+6i^2}{1-i^2}$$

$$= \frac{-4-2i-6}{1-(-1)} = \frac{-10-2i}{2} = -5-i$$

To represent this complex number graphically, we plot the point $(-5,-1)$ on the complex plane, where the $x-$axis represents the real part and the $y-$axis represents the imaginary part.