Exercise 1.1

Q1. Simplify and express in terms of iy.

i. $\sqrt{-3}$

Solution: $\sqrt{-3}$

Since $\sqrt{-1} = i$, we have:

$\Rightarrow$ $\sqrt{-3} = \sqrt{3 \cdot (-1)} = \sqrt{3} \cdot \sqrt{-1} = \sqrt{3}i$; $[\because \ i = \sqrt{-1}]$

ii. $6\sqrt{-4}$

Solution: $6\sqrt{-4}$

$\Rightarrow$ $6\sqrt{-4} = 6\sqrt{4 \cdot (-1)} = 6 \cdot \sqrt{4} \cdot \sqrt{-1} = 6 \cdot 2 \cdot i = 12i$; $[\because \ i = \sqrt{-1}]$

iii. $\sqrt{-\frac{4}{9}}$

Solution: $\sqrt{-\frac{4}{9}}$

$\Rightarrow$ $\sqrt{-\frac{4}{9}} = \sqrt{\frac{4}{9} \cdot (-1)} = \sqrt{\frac{4}{9}} \cdot \sqrt{-1} = \frac{2}{3}i$; $[\because \ i = \sqrt{-1}]$

iv. $-\sqrt{-20}$

Solution: $-\sqrt{-20}$

$\Rightarrow$ $-\sqrt{-20} = -\sqrt{20 \cdot (-1)} = -\sqrt{20} \cdot \sqrt{-1}$

$= \sqrt{4} \cdot \sqrt{5} \cdot \sqrt{-1} = 2\sqrt{5} \cdot \sqrt{-1} = -2\sqrt{5}i$

v. $4 - \sqrt{-60}$

Solution: $4 - \sqrt{-60}$.

$\Rightarrow$ $4 - \sqrt{-60} = 4 - \sqrt{60 \cdot (-1)} = 4 - \sqrt{60} \cdot \sqrt{-1} = 4 - \sqrt{4} \cdot \sqrt{15} \cdot \sqrt{-1}$

$= 4 - \sqrt{4} \cdot \sqrt{15} \cdot \sqrt{-1} = 4 - 2\sqrt{15}i$

vi. $\sqrt{-8}\sqrt{-2}$

Solution: $\sqrt{-8}\sqrt{-2}$

$\Rightarrow$ $\sqrt{-8}\sqrt{-2} = (\sqrt{8}\cdot\sqrt{-1})(\sqrt{2}\cdot\sqrt{-1}) = (\sqrt{8}i)(\sqrt{2}i)$

$= \sqrt{8}\cdot\sqrt{2}\cdot i^2 = \sqrt{16}\cdot i^2 = 4\cdot(-1) = -4 \quad ; \quad [\because \ i^2 = -1]$

Q2. <u>Simplify:</u>

i. $(4-i)+(5+5i)$

Solution: $(4-i)+(5+5i)$

We can combine the real parts and the imaginary parts separately:

$\Rightarrow$ $(4-i)+(5+5i) = (4+5)+(-i+5i) = 9+4i$

ii. $(7-6i)-(5-6i)$

Solution: $(7-6i)-(5-6i)$

We can combine the real parts and the imaginary parts separately:

$\Rightarrow$ $(7-6i)-(5-6i) = 7-6i-5+6i$

Therefore, the simplified expression is $2+0i = 2$.

iii. $(-2+8i)-(7+3i)$

Solution: $(-2+8i)-(7+3i)$

We can combine the real parts and the imaginary parts separately:

$\Rightarrow$ $(-2+8i)-(7+3i) = -2+8i-7-3i = -9+5i$

iv. $(4-2i)-(5-2i)$

Solution: $(4-2i)-(5-2i)$

We can combine the real parts and the imaginary parts separately:

$\Rightarrow$ $(4-2i)-(5-2i) = 4-2i-5+2i-1+0i = -1$

v. $(2+4i)(1+2i)$

Solution: $(2+4i)(1+2i)$

$\Rightarrow$ $(2+4i)(1+2i) = 2(1)+2(2i)+4i(1)+4i(2i) = 2+4i+4i+8i^2$

Since $i^2 = -1$, we can substitute that into the expression:

$\Rightarrow$ $2+4i+4i+8(-1) = 2+8i-8$

Now, combine the real and imaginary parts:

$\Rightarrow$ $(2-8)+8i = -6+8i$

vi. $(1-4i)(2-3i)$

Solution: $(1-4i)(2-3i)$

$\Rightarrow$ $(1-4i)(2-3i) = 1(2)+1(-3i)-4i(2)-4i(-3i)$

$= 2-3i-8i+12i^2$

Since $i^2 = -1$, we can substitute that into the expression:

$\Rightarrow$ $2-3i-8i+12(-1) = 2-11i-12$

Now, combine the real and imaginary parts:

$\Rightarrow$ $(2-12)-11i = -10-11i$

vii. $-8i(2-2i)$

Solution: $-8i(2-2i)$

$\Rightarrow$ $-8i(2-2i) = -8i(2) - 8i(-2i) = -16i + 16i^2$

Since $i^2 = -1$, we can substitute that into the expression:

$\Rightarrow$ $-16i + 16(-1) = -16i - 16$

Now, rewrite the expression in the standard form $+bi$:

$= -16 - 16i$

viii. $(3+2i)^2$

Solution: $(3+2i)^2$

$\Rightarrow$ $(3+2i)^2 = (3+2i)(3+2i)$

$\Rightarrow$ $(3+2i)(3+2i) = 3(3) + 3(2i) + 2i(3) + 2i(2i) = 9 + 6i + 6i + 4i^2$

Since $i^2 = -1$, we can substitute that into the expression:

$\Rightarrow$ $9 + 6i + 6i + 4(-1) = 9 + 12i - 4$

Now, combine the real and imaginary parts:

$\Rightarrow$ $(9-4) + 12i = 5 + 12i$

ix. $(3-6i)(3+6i)$

Solution: $(3-6i)(3+6i)$

$\Rightarrow$ $(3-6i)(3+6i) = 3(3) + 3(6i) - 6i(3) - 6i(6i)$

$= 9 + 18i - 18i - 36i^2$

Since $i^2 = -1$, we can substitute that into the expression:

$\Rightarrow$ $9 + 18i - 18i - 36(-1) = 9 - 36(-1) = 9 + 36 = 45$

x. $(-5-3i)^2$

Solution: $(-5-3i)^2$

$\Rightarrow$ $(-5-3i)^2 = (-5-3i)(-5-3i)$

$\Rightarrow$ $(-5-3i)(-5-3i) = (-5)(-5) + (-5)(-3i) + (-3i)(-5) + (-3i)(-3i)$

$= 25 + 15i + 15i + 9i^2$

Since $i^2 = -1$, we can substitute that into the expression:

$\Rightarrow$ $25 + 15i + 15i + 9(-1) = 25 + 30i - 9$

Now, combine the real and imaginary parts:

$\Rightarrow$ $(25-9) + 30i = 16 + 30i$

xi. $(1+\sqrt{2}i)(1-\sqrt{3}i)$

Solution: $(1+\sqrt{2}i)(1-\sqrt{3}i)$

$\Rightarrow$ $(1+\sqrt{2}i)(1-\sqrt{3}i) = 1(1) + 1(-\sqrt{3}i) + \sqrt{2}i(1) + \sqrt{2}i(-\sqrt{3}i)$

$= 1 - \sqrt{3}i + \sqrt{2}i - \sqrt{6}i^2$

Since $i^2 = -1$, we can substitute that into the expression:

$\Rightarrow$ $1 - \sqrt{3}i + \sqrt{2}i - \sqrt{6}(-1) = 1 - \sqrt{3}i + \sqrt{2}i + \sqrt{6}$

Now, combine the real and imaginary parts:

$\Rightarrow$ $(1+\sqrt{6}) + (-\sqrt{3}+\sqrt{2})i = (1+\sqrt{6}) + (\sqrt{2}-\sqrt{3})i$

xii. $(\sqrt{2}+i)(\sqrt{2}-i)$

Solution: $(\sqrt{2}+i)(\sqrt{2}-i)$

$\Rightarrow \quad (\sqrt{2}+i)(\sqrt{2}-i) = (\sqrt{2})(\sqrt{2}) + (\sqrt{2})(-i) + i(\sqrt{2}) + i(-i)$

$= 2 - \sqrt{2}i + \sqrt{2}i - i^2$

Since $i^2 = -1$, we can substitute that into the expression:

$\Rightarrow \quad 2 - \sqrt{2}i + \sqrt{2}i - (-1) = 2 + 1 = 3$

Q3. <u>Simplify:</u>

i. i^9

Solution: i^9

$\Rightarrow \quad i^9 = i^8 \cdot i = (i^2)^4 \cdot i^1$

Since $i^2 = -1$, we have:

$\Rightarrow \quad i^9 = (-1)^4 \cdot i = 1 \cdot i = i$

ii. i^{13}

Solution: i^{13}

$\Rightarrow \quad i^{13} = i^{12} \cdot i = (i^2)^6 \cdot i^1$

Since $i^2 = -1$, we have:

$\Rightarrow \quad i^{13} = (-1)^6 \cdot i = 1 \cdot i = i$

iii. i^{28}

Solution: i^{28}

$\Rightarrow \quad i^{28} = (i^2)^{14}$

Since $i^2 = -1$, we have:

$\Rightarrow \quad i^{28} = (-1)^{14} = 1$

iv. $(-i)^{21}$

Solution: $(-i)^{21}$

$\Rightarrow \quad (-i)^{21} = (-1)^{21} \cdot (i)^{21}$

Since $(-1)^{21} = -1$, we have:

$\Rightarrow \quad (-i)^{21} = -i^{21}$

Now, we need to simplify i^{21}. We can express i^{21} as follows:

$\Rightarrow \quad -i^{21} = -i^{20} \cdot i = -(i^2)^{10} \cdot i^1$

Since $i^2 = -1$, we have:

$= -(-1)^{10} \cdot i = -1 \cdot i = -i$

v. $(3i)^3$

Solution: $(3i)^3$

$= 27 \cdot i^3$

Substitute the simplified values: Since $i^2 = -1$, we have:

$\Rightarrow \quad 27 \cdot i^3 = 27i^2 \cdot i = 27 \cdot (-1) \cdot i = -27i$

vi. $(-2i)^4$

Solution: $(-2i)^4$

$(-2i)^4 = 16 \cdot i^4 = 16(i^2)^2 = 16(-1)^2 = 16$; $[\because \quad i^2 = -1]$

Q4. <u>Simplify in the form of $a + bi$.</u>

i. $9 + i^6$

Solution: $9 + i^6$

$\Rightarrow \quad 9 + i^6 = 9 + (i^2)^3 = 9 + (-1)^3 = 9 - 1 = 8$; $[\because \quad i^2 = -1]$

Since there is no imaginary part, we can write this as $8 + 0i$.

ii. $-17 + i^5$

Solution: $-17 + i^5$

$\Rightarrow \quad -17 + i^5 = -17 + i^4 \cdot i = -17 + (i^2)^2 \cdot i$

$= -17 + (-1)^2 \cdot i = -17 + i$; $[\because \quad i^2 = -1]$

iii. $-13i$

Solution: $i^4 - 13i$

$\Rightarrow \quad i^4 - 13i = (i^2)^2 - 13i = (-1)^2 - 13i = 1 - 13i$; $[\because \quad i^2 = -1]$

iv. $i^5 + 21i$

Solution: $i^5 + 21i$

$\Rightarrow \quad i^5 + 21i = i^4 \cdot i + 21i = (i^2)^2 \cdot i + 21i = (-1)^2 \cdot i + 21i$

$= 1 \cdot i + 21i = i + 21i = 22i$; $[\because \quad i^2 = -1]$

Since there is no real part, we can write this as $0 + 22i$.

v. $i^5 + i^7$

Solution: $i^5 + i^7$

$\Rightarrow \quad i^5 + i^7 = i^4 \cdot i + i^6 \cdot i = (i^2)^2 \cdot i + (i^2)^3 \cdot i$; $[\because \quad i^2 = -1]$

$= (-1)^2 \cdot i + (-1)^3 \cdot i = i + (-i) = i - i = 0$

Since there is no imaginary part, we can write this as $0 + 0i$.

vi. $i^{74} - i^{100}$

Solution: $i^{74} - i^{100}$

$\Rightarrow \quad i^{74} - i^{100} = (i^2)^{37} - (i^2)^{50} = (-1)^{37} - (-1)^{50}$; $[\because \quad i^2 = -1]$

$= -1 - 1 = -2$

Since there is no imaginary part, we can write this as $-2 + 0i$.

Q5. <u>Divide and simplify in the form of $a + bi$.</u>

i. $\dfrac{3}{4 - i}$

Solution: $\dfrac{3}{4 - i}$

We need to multiply the numerator and denominator by the conjugate of the denominator. The conjugate of $4 - i$ is $4 + i$.

$$\Rightarrow \quad \frac{3}{4-i} = \frac{3}{4-i} \times \frac{4+i}{4+i} = \frac{3(4+i)}{(4-i)(4+i)} = \frac{12+3i}{(4)^2-i^2}$$

Since $i^2 = -1$, we can substitute this into the denominator:

$$\Rightarrow \quad \frac{12+3i}{16-(-1)} = \frac{12+3i}{16+1} = \frac{12+3i}{17}$$

Finally, separate the real and imaginary parts:

$$\Rightarrow \quad \frac{12+3i}{17} = \frac{12}{17} + \frac{3}{17}i$$

ii. $\frac{3i}{6+5i}$

Solution: $\frac{3i}{6+5i}$

We need to multiply the numerator and denominator by the conjugate of the denominator. The conjugate of $6+5i$ is $6-5i$.

$$\Rightarrow \quad \frac{3i}{6+5i} = \frac{3i}{6+5i} \times \frac{6-5i}{6-5i} = \frac{3i(6-5i)}{(6+5i)(6-5i)}$$

$$\Rightarrow \quad \frac{3i(6-5i)}{(6+5i)(6-5i)} = \frac{18i-15i^2}{(6)^2-25i^2}$$

Since $i^2 = -1$, we can substitute this into the numerator and denominator:

$$\Rightarrow \quad \frac{18i-15(-1)}{36-25(-1)} = \frac{18i+15}{36+25} = \frac{15+18i}{61}$$

Finally, separate the real and imaginary parts:

$$\Rightarrow \quad \frac{15+18i}{61} = \frac{15}{61} + \frac{18}{61}i$$

iii. $\frac{3-i\sqrt{5}}{3+i\sqrt{5}}$

Solution: $\frac{3-i\sqrt{5}}{3+i\sqrt{5}}$

We need to multiply the numerator and denominator by the conjugate of the denominator. The conjugate of $3+i\sqrt{5}$ is $3-i\sqrt{5}$.

$$\Rightarrow \quad \frac{3-i\sqrt{5}}{3+i\sqrt{5}} = \frac{3-i\sqrt{5}}{3+i\sqrt{5}} \times \frac{3-i\sqrt{5}}{3-i\sqrt{5}} = \frac{(3-i\sqrt{5})^2}{(3+i\sqrt{5})(3-i\sqrt{5})}$$

Now, multiply out the numerator and denominator:

$$\Rightarrow \quad \frac{(3-i\sqrt{5})^2}{(3+i\sqrt{5})(3-i\sqrt{5})} = \frac{(3)^2 - 2\times 3\times i\sqrt{5} + (i\sqrt{5})^2}{(3)^2-(i\sqrt{5})^2}$$

$$= \frac{9-6i\sqrt{5}+5i^2}{9-(-5i^2)} = \frac{9-6i\sqrt{5}-5}{9-(-5i^2)} = \frac{4-6i\sqrt{5}}{14} \quad ; \qquad [\because \; i^2 = -1]$$

Finally, separate the real and imaginary parts:

$$\Rightarrow \quad \frac{4-6i\sqrt{5}}{14} = \frac{4}{14} - \frac{6\sqrt{5}}{14}i = \frac{2}{7} - \frac{3\sqrt{5}}{7}i$$

iv. $\dfrac{2+7i}{5i}$

Solution: $\dfrac{2+7i}{5i}$

We need to multiply the numerator and denominator by the conjugate of the denominator. The conjugate of $5i$ is $-5i$.

$$\Rightarrow \quad \frac{2+7i}{5i} = \frac{2+7i}{5i} \times \frac{-5i}{-5i} = \frac{(2+7i)(-5i)}{(5i)(-5i)}$$

Now, multiply out the numerator and denominator:

$$\Rightarrow \quad \frac{(2+7i)(-5i)}{(5i)(-5i)} = \frac{-10i - 35i^2}{-25i^2}$$

Since $i^2 = -1$, we can substitute this into the numerator and denominator:

$$\Rightarrow \quad \frac{-10i - 35(-1)}{-25(-1)} = \frac{-10i + 35}{25} = \frac{35 - 10i}{25}$$

Finally, separate the real and imaginary parts:

$$\Rightarrow \quad \frac{35 - 10i}{25} = \frac{35}{25} - \frac{10}{25}i = \frac{7}{5} - \frac{2}{5}i$$

v. $\dfrac{4+5i}{4-5i}$

Solution: $\dfrac{4+5i}{4-5i}$

We need to multiply the numerator and denominator by the conjugate of the denominator. The conjugate of $4-5i$ is $4+5i$.

$$\Rightarrow \quad \frac{4+5i}{4-5i} = \frac{4+5i}{4-5i} \times \frac{4+5i}{4+5i} = \frac{(4+5i)^2}{(4-5i)(4+5i)}$$

Now, multiply out the numerator and denominator:

$$\Rightarrow \quad \frac{(4+5i)^2}{(4-5i)(4+5i)} = \frac{(4)^2 + 40i + 25i^2}{(4)^2 - 25i^2}$$

Since $i^2 = -1$, we can substitute this into the numerator and denominator:

$$\Rightarrow \quad \frac{16 + 40i + 25(-1)}{16 - 25(-1)} = \frac{16 + 40i - 25}{16 + 25} = \frac{-9 + 40i}{41}$$

Finally, separate the real and imaginary parts:

$$\Rightarrow \quad \frac{-9 + 40i}{41} = -\frac{9}{41} + \frac{40}{41}i$$

vi. $\dfrac{3+2i}{2+i}$

Solution: $\dfrac{3+2i}{2+i}$

We need to multiply the numerator and denominator by the conjugate of the denominator. The conjugate of $2+i$ is $2-i$.

$$\Rightarrow \quad \frac{3+2i}{2+i} = \frac{3+2i}{2+i} \times \frac{2-i}{2-i} = \frac{(3+2i)(2-i)}{(2+i)(2-i)}$$

Now, multiply out the numerator and denominator:

$$\Rightarrow \quad \frac{(3+2i)(2-i)}{(2+i)(2-i)} = \frac{6-3i+4i-2i^2}{(2)^2-i^2}$$

Since $i^2 = -1$, we can substitute this into the numerator and denominator:

$$\Rightarrow \quad \frac{6-3i+4i-2(-1)}{4-(-1)} = \frac{6+i+2}{4+1} = \frac{8+i}{5}$$

Finally, separate the real and imaginary parts:

$$\Rightarrow \quad \frac{8+i}{5} = \frac{8}{5} + \frac{1}{5}i$$

vii. $\frac{5+i}{1+2i}$

Solution: $\frac{5+i}{1+2i}$

We need to multiply the numerator and denominator by the conjugate of the denominator. The conjugate of $1+2i$ is $1-2i$.

$$\Rightarrow \quad \frac{5+i}{1+2i} = \frac{5+i}{1+2i} \times \frac{1-2i}{1-2i} = \frac{(5+i)(1-2i)}{(1+2i)(1-2i)}$$

Now, multiply out the numerator and denominator:

$$\Rightarrow \quad \frac{(5+i)(1-2i)}{(1+2i)(1-2i)} = \frac{5-10i+i-2i^2}{(1)^2-4i^2}$$

Since $i^2 = -1$, we can substitute this into the numerator and denominator:

$$\Rightarrow \quad \frac{5-9i-2(-1)}{1-4(-1)} = \frac{5-9i+2}{1+4} = \frac{7-9i}{5}$$

Finally, separate the real and imaginary parts:

$$\Rightarrow \quad \frac{7-9i}{5} = \frac{7}{5} - \frac{9}{5}i$$

viii. $\frac{a+ib}{a-ib}$

Solution: $\frac{a+ib}{a-ib}$

We multiply the numerator and the denominator by the conjugate of the denominator, which is $a+ib$.

$$\Rightarrow \quad \frac{a+ib}{a-ib} = \frac{(a+ib)(a+ib)}{(a-ib)(a+ib)}$$

$$= \frac{a^2+2abi+(ib)^2}{a^2-(ib)^2} = \frac{a^2+2abi+i^2b^2}{a^2-i^2b^2}$$

Since $i^2 = -1$, we have:

$\Rightarrow \quad \frac{a^2+2abi-b^2}{a^2+b^2}=\frac{(a^2-b^2)+2abi}{a^2+b^2}$

Finally, separate the real and imaginary parts:

$=\frac{a^2-b^2}{a^2+b^2}+\frac{2ab}{a^2+b^2}i$

ix. $\frac{1+i}{(1-i)^2}$

Solution: $\frac{1+i}{(1-i)^2}$ ……… (1)

First, we expand the denominator:

$\Rightarrow \quad (1-i)^2=(1-i)(1-i)=1-i-i+i^2=1-2i+i^2$

Since $i^2=-1$, we have:

$\Rightarrow \quad (1-i)^2=1-2i-1=-2i$

So the expression (1) becomes: $\frac{1+i}{-2i}$

Now, we multiply the numerator and the denominator by the conjugate of the denominator, which is:

$\Rightarrow \quad \frac{1+i}{-2i}=\frac{(1+i)(i)}{(-2i)(i)}=\frac{i+i^2}{-2i^2} \quad \Rightarrow \quad \frac{i-1}{-2(-1)}=\frac{i-1}{\cdot 2}=\frac{-1+i}{2} \quad ; \quad [\because\ i^2=-1]$

Finally, separate the real and imaginary parts:

$=-\frac{1}{2}+\frac{1}{2}i$

x. $\frac{(2+2i)^2}{(1+i)^2}$

Solution: $\frac{(2+2i)^2}{(1+i)^2}$ ……… (1)

First, we expand the numerator and the denominator separately:

$\Rightarrow \quad (2+2i)^2=(2+2i)(2+2i)=4+4i+4i+4i^2=4+8i+4i^2$

Since $i^2=-1$, we have:

$\Rightarrow \quad (2+2i)^2=4+8i-4=8i$

$\Rightarrow \quad (1+i)^2=(1+i)(1+i)=1+i+i+i^2=1+2i+i^2$

Since $i^2=-1$, we have:

$\Rightarrow \quad (1+i)^2=1+2i-1=2i$

So the expression (1) becomes: $\frac{8i}{2i}$

Now, we can simplify the fraction by canceling the common factor $2i$.

$\Rightarrow \quad \frac{8i}{2i}=\frac{8}{2}=4$

Thus, the simplified form is $4+0i$.

❖ ❖ ❖ ❖ ❖